

Math 504 Set Theory I
Problem Set 8

Due Monday April 14

1) Suppose $\tau \in \mathcal{M}^{\mathbb{P}}$ and $\tau \subseteq \{\check{n} : n \in \omega\} \times \mathbb{P}$. Let

$$\sigma = \{\langle \check{n}, p \rangle : \forall q \in \mathbb{P} (\langle \check{n}, q \rangle \in \tau \rightarrow p \perp q)\}.$$

Show that $\sigma_G = \omega \setminus \tau_G$ for any \mathcal{M} -generic filter $G \subseteq \mathbb{P}$.

2) Let $D \subseteq \mathbb{P}$ and $p \in \mathbb{P}$. Prove that the following are equivalent.

- i) D is dense below p .
- ii) for all $q \leq p$, D is dense below q .
- iii) $\{q \leq p : D \text{ is dense below } q\}$ is dense below p .

3) Prove that the following are equivalent:

- i) $p \Vdash^* \phi(\tau_1, \dots, \tau_n)$
- ii) $\{q : q \Vdash^* \phi(\tau_1, \dots, \tau_n)\}$ is dense below p .