MATH 531: PROBLEM SET 3

Due Friday, September 19

(1) Let $R \subset S$ be a ring extension, and let $\tilde{R}$ be the integral closure of R in S.

(a) Show that $\tilde{R}[X]$ is the integral closure of $R[X]$ in $S[X]$.

(b) Show that R is integrally closed in S if and only if $R[X]$ is integrally closed in $S[X]$.

(c) Let R be a domain. Show that R is normal if and only if $R[X]$ is normal.

(2) For each $n \in \mathbb{Z}$, find the integral closure of $\mathbb{Z}[\sqrt{n}]$. (You can use the strategy suggested in Eisenbud's book, p.138, if you wish.)

(3) Show that if R is a valuation ring and $P \subset R$ is a prime ideal, then R_P and R/P are valuation rings.

(4) Let R be a valuation ring. The following are equivalent:

(1) R is Noetherian.
(2) R is a PID.