

## MATH 531: PROBLEM SET 4

Due Friday, September 26

(1) Atiyah-MacDonald, Ch. 5: 30, 33; Ch.9: 4.

(2) (Matsumura, 10.11) Let  $k$  be a field,  $X$  and  $Y$  indeterminates, and suppose  $\alpha$  is a positive irrational number. Then the map

$$v : k[X, Y] \rightarrow \mathbb{R} \cup \{\infty\}, \quad \sum c_{n,m} X^n Y^m \rightarrow \min\{n + m\alpha \mid c_{n,m} \neq 0\}$$

determines a valuation of  $k(X, Y)$  with value group  $\mathbb{Z} + \mathbb{Z}\alpha$ .

(3) Let  $R$  be the ring of integers in an algebraic number field  $K$  (i.e.  $K$  is a finite algebraic extension of  $\mathbb{Q}$ , and  $R$  is the integral closure of  $\mathbb{Z}$  in  $K$ ). Show that the localization of  $R$  at every prime ideal is a DVR. (Hint: read and use 5.17 in Atiyah-MacDonald.)