MATH 531: PROBLEM SET 7

Due Friday, October 24

(1) Let (R, m) be a Noetherian local ring, and suppose that the elements $a_1, \ldots, a_t$ are part of a system of parameters. If the ideal $P = (a_1, \ldots, a_t)$ is prime and has height t, show that $\mathrm{ht}\ P + \mathrm{coht}\ P = \dim R$.

(2) Let $S = k[[X, Y, Z]]$, with k a field, and let $R = S/I$, where $I = (XY, XZ)$.

(a) Find the minimal prime ideals in R. Find the dimension of R.

(b) Show that $\{\overline{Z}, \overline{X}+\overline{Y}\}$ is a system of parameters for R, but not an R-regular sequence.

(c) Show that $\mathrm{ht}\ (Z) = 0$. (This shows that if $\{a_1, \ldots, a_r\}$ is a system of parameters, it is not necessarily the case that $\mathrm{ht}\ (a_1, \ldots, a_i) = i$.)

(3) Let $R = k[X, Y]/(Y^2 - X^3)$, where k is a field.

(a) If $\mathrm{m} = (\overline{X}, \overline{Y})$, show that R_{m} is not a regular local ring. Compute the height and the depth of its maximal ideal.

(b) If char $k \neq 2, 3$ and $\mathrm{m} = (\overline{X} - 1, \overline{Y} - 1)$, show that R_{m} is a regular local ring and compute its dimension.

(4) Show that any localization of the ring R in problem 3 at a maximal ideal is Cohen-Macaulay.

For the next two problems you can assume the following fact that will be proved later in the course: if R is a Noetherian local ring and $x \in R$ is a non-zero divisor, then depth $R/xR =$ depth $R - 1$. (In both problems one can replace the power series ring with the localization of the polynomial ring at the maximal ideal generated by the variables.)

(5) Show that the ring $R = k[[X, Y, Z]]/(XY, XZ)$ in problem 2 is not Cohen-Macaulay.

(6) Show that the ring $R = k[[X, Y, Z, T]]/(X, Y) \cap (Z, T)$ is not Cohen-Macaulay.