MATH 531: PROBLEM SET 8

Due Friday, October 31

(1) Show that a chain morphism $f : C_\bullet \to D_\bullet$ takes cycles to cycles and boundaries to boundaries. Deduce that it induces a homomorphism $H_n(f) : H_n(C_\bullet) \to H_n(D_\bullet)$.

(2) Show that if f and g are homotopic maps between complexes $C_\bullet$ and $D_\bullet$, then $H_n(f) = H_n(g)$.

(3) Show that a module is projective if and only if it is a direct summand of every module of which it is a quotient.

(4) Let R be an integral domain with quotient field $Q(R)$. Show that if M is a vector space over $Q(R)$, then M is a divisible R-module. Conversely, show that if M is a torsion-free divisible R-module, then M is a vector space over $Q(R)$.

(5) If R is an integral domain that is not a field, then $\mathrm{Hom}_R(Q(R), R) = 0$. (In particular $\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Q}, \mathbb{Z}) = 0$.)

(6) Show that $\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}_2, \mathbb{Q}) = 0$, but $\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}_2, \mathbb{Q}/\mathbb{Z}) \neq 0$.

(7) Show that the functor $\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}_2, \bullet)$ is not right exact.

(8) Show that the functor $\mathrm{Hom}_{\mathbb{Z}}(\bullet, \mathbb{Z})$ is not right exact.