

Sample Midterm Math 535 Spring 2006

1. Find

$$\int_{|z|=4} \frac{z}{z^2 + 2z + 1} dz.$$

2. Show that if an analytic function $w = u + iv = f(z)$ takes all its values on the hyperbola $v = \frac{1}{u}$ then f is a constant

3. a)

Find the domain of convergence of the power series

$$\sum_{n=0}^{\infty} 2^{n/2} z^n.$$

- b) If $f(z) = \sum a_n z^n$ find a formula in terms of f for $\sum n^2 a_n z^{n+1}$.

4. Compute $\int_C e^z dz$ where C is the semicircle in the upper half plane joining -3 to 3 and show it is real.

Hint: Is there another path you can use for the computation?

5. Show that the image of an entire function $f(z)$ is dense in the complex plane.

Hint: Assuming not, this means that $f(z)$ misses an entire disc $D(z_0, \delta)$. Find some other function in terms of $f(z)$ to which you can apply the Liouville theorem to get a contradiction.