HOMEWORK 11

This problem set is due Monday November 17. You may work on the problem set in groups; however, the final write-up must be yours and reflect your own understanding.

Problem 0.1. *Let $E : y^2 = x^3 + ax + b$ be a non-singular cubic curve. If $P \in E$ has coordinates (x, y), calculate the x-coordinate of $2P$. Find the points of order two on a non-singular cubic curve in Weierstrass normal form.*

Problem 0.2. *Describe the points of order three on a non-singular curve in Weierstrass normal form. Deduce that a non-singular cubic has nine flex lines. Show that the line joining any two inflection points intersects the curve in a third inflection point. Hence, the inflection points of a non-singular cubic are nine non-collinear points such that a line joining any two contains a third. Is it possible to have a set S of finitely many non-collinear real points in $\mathbb{R}^2$ such that the line joining any two contains another point of S?*

Problem 0.3. *Let $E : y^2 = x^3 + ax + b$ be a non-singular cubic with a and b in a number field k. Prove that the k-points $E(k)$ form a subgroup of $E(\mathbb{C})$.*

Problem 0.4. *Show that the equation $y^2 = x^3 - 2$ has infinitely many rational solutions.*