

p-ADIC INTEGRATION EXERCISES

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Problem 0.1. Let χ be an unramified quasi-character. Then for any k , we have

$$\int_{|t|>q^k} \chi(t) dt = (1 - q^{-1}) \frac{\chi(\varpi)^{-1}q}{1 - \chi(\varpi)^{-1}q} \chi(\varpi)^{-k} q^k.$$

Problem 0.2. Let ψ be an unramified additive character. Then prove that we have

$$\int_U \psi(\varpi^{-n}\epsilon) d\epsilon = \begin{cases} -q^{-1} & n = 1; \\ 0 & n > 1. \end{cases}$$

Problem 0.3. For an unramified quasi-character η and ψ an unramified additive character define

$$J(\eta, y) := \int_{|s|>1} \eta(s)\psi(ys) ds.$$

Then prove that if y is not integral, $J(\eta, y) = 0$. If y is integral, then prove that

$$J(\eta, y) = (1 - \eta(\varpi)^{-1}) \frac{\eta(\varpi)^{-1}q}{\eta(\varpi)^{-1}q - 1} \eta(y)^{-1} |y|^{-1} - (1 - q^{-1}) \frac{\eta(\varpi)^{-1}q}{\eta(\varpi)^{-1}q - 1}.$$

When y is a unit, verify that the above expression simplifies to

$$J(\eta, y) = -\eta(\varpi)^{-1}.$$

Problem 0.4. For an unramified quasi-character κ , prove that we have

$$\int_U \kappa(1 + v) dv = \frac{1 - 2q^{-1} + \kappa(\varpi)q^{-2}}{1 - \kappa(\varpi)q^{-1}}$$

Problem 0.5. For an unramified quasi-character κ , set

$$I(\kappa) = \int_{|u|<1, |v|<1} \kappa(u + v) du dv$$

Then prove that

$$I(\kappa) = \frac{\kappa(\varpi)q^{-2}(1 - q^{-1})}{1 - \kappa(\varpi)q^{-1}}.$$