FIRST HOMEWORK SET, MA 430, DUE SEPTEMBER 2ND

1. Exercise

Formalise the following arguments in propositional logic.

(a) If investment banks offer large compensation to their traders, then people will be outraged, but the traders will be rich. If the traders will be rich, their kids will be happy. If people will be outraged, Obama will not be reelected. If banks do not offer large compensation to their traders, then traders will quit. Therefore, either Obama will not be reelected and traders' kids will be happy, or traders will quit.

(b) If capital investment remains constant, then government spending will not increase or unemployment will result. If government spending will not increase, taxes can be reduced. If taxes can be reduced and capital investment remains constant, then unemployment will not result. Hence government spending will increase.

2. Exercise

Write the tree decomposition of the following formulas and calculate their height:

(1) $((P \leftrightarrow Q) \rightarrow \neg(\neg S \vee R))$,
(2) $(R \rightarrow \neg(S \leftrightarrow (P \wedge Q)))$,
(3) $\neg(\neg A \rightarrow \neg(\neg(R \rightarrow \neg Q) \vee S))$.

3. Exercise

Write the truth tables for the following formulas

(1) $((P \leftrightarrow Q) \rightarrow \neg(\neg Q \vee R))$,
(2) $(P \rightarrow \neg(Q \leftrightarrow (P \wedge Q)))$,
(3) $\neg(\neg Q \rightarrow \neg(\neg(R \rightarrow \neg Q) \vee S))$.

4. Exercise

This exercise introduces so called *Polish notation*, which allows us to write formulas without using parentheses.

Suppose L is a propositional language and let the set of well-formed formulas in Polish notation, $\mathrm{Polish(L)}$, be the smallest set of expressions such that

(1) any propositional variable belongs to $\mathrm{Polish(L)}$,
(2) if $A \in \mathrm{Polish(L)}$, then $\neg A \in \mathrm{Polish(L)}$,
(3) if $A, B \in \mathrm{Polish(L)}$, then $\vee AB, \wedge AB, \rightarrow AB$, and $\leftrightarrow AB \in \mathrm{Polish(L)}$.

(a) Using the same interpretation of the logical connectives, rewrite the following formulas in Polish notation:

(1) $((P \leftrightarrow Q) \rightarrow \neg(\neg S \vee R))$,
(2) $(R \rightarrow \neg(S \leftrightarrow (P \wedge Q)))$,
(3) $\neg(\neg A \rightarrow (R \vee S))$.

(b) And vice versa, rewrite the following formulas in Polish notation into ordinary notation:

(1) $\neg \rightarrow \vee RSP$,
(2) $\vee \rightarrow RS \wedge \neg PQ$,
(3) $\leftrightarrow \vee \wedge RP \neg SP$.

(c) If we count each of $\vee, \wedge, \rightarrow, \leftrightarrow$ as $+1$, $\neg$ as 0, and each propositional variable as -1, prove that an expression A written using only propositional variables and logical connectives is a well-formed formula in Polish notation if and only if (i) the sum of symbols in A is -1 and (ii) the sum of the symbols in any initial segment of the expression A is non-negative. [Hint: One direction is proved by induction on the number of symbols in A.]

(d) Use the criterion from (c) to decide which ones of the following expressions are well-formed formulas in Polish notation:

(1) $\rightarrow \neg SR \rightarrow P$,
(2) $\vee \rightarrow \leftrightarrow RPS \rightarrow \leftrightarrow QP$,
(3) $\vee \rightarrow \wedge \neg R \rightarrow SPRQ$.