7. In all of the languages considered in this exercise, \( R \) is a binary relation symbol, \( * \) and \( \oplus \) are binary function symbols, \( c \) and \( d \) are constant symbols.

We will write \( x \oplus y \) and \( x * y \) respectively, rather than \( \oplus xy \) and \( *xy \) (with a reminder that this necessitates the use of parentheses when writing terms). \( x^2 \) will be an abbreviation for \( x * x \).

(a) In each of the following six cases (1 \( \leq \) \( i \) \( \leq \) 6), a language \( L_i \) and two \( L_i \)-structures \( A_i \) and \( B_i \) are given and you are asked to find a closed formula of \( L_i \) that is true in \( A_i \) and false in \( B_i \).

\[
\begin{align*}
(1) & \quad L_1 = \{ R \} \quad A_1 = \langle \mathbb{N}, \leq \rangle \quad B_1 = \langle \mathbb{Z}, \leq \rangle \\
(2) & \quad L_2 = \{ R \} \quad A_2 = \langle \mathbb{Q}, \leq \rangle \quad B_2 = \langle \mathbb{Z}, \leq \rangle \\
(3) & \quad L_3 = \{ * \} \quad A_3 = \langle \mathbb{N}, x \rangle \quad B_3 = \langle \wp(\mathbb{N}), \cap \rangle \\
(4) & \quad L_4 = \{ c, * \} \quad A_4 = \langle \mathbb{N}, 1, x \rangle \quad B_4 = \langle \mathbb{Z}, 1, x \rangle \\
(5) & \quad L_5 = \{ c, d, \oplus, * \} \quad A_5 = \langle \mathbb{R}, 0, 1, +, x \rangle \quad B_5 = \langle \mathbb{Q}, 0, 1, +, x \rangle \\
(6) & \quad L_6 = \{ R \} \quad A_6 = \langle \mathbb{Z}, \equiv_2 \rangle \quad B_6 = \langle \mathbb{Z}, \equiv_3 \rangle \\
\end{align*}
\]

\( x \) and \( + \) are the usual operations of multiplication and addition, \( \cap \) is the operation of intersection, \( \equiv_p \) is the relation of congruence modulo \( p \).

(b) For each of the following closed formulas of the language \( \{ c, \oplus, *, R \} \), find a model of the formula as well as a model of its negation.

\[
\begin{align*}
F_1 : \quad & \forall u \forall v \exists x (\neg v \equiv c \Rightarrow u \oplus (v * x) \equiv c) \\
F_2 : \quad & \forall u \forall v \forall w \exists x (\neg w \equiv c \Rightarrow u \oplus (v * x) \oplus (w * x^2) \equiv c)
\end{align*}
\]