Exercise 1

Show that the following argument is valid using the tableau method:

\[ \forall x (\exists y Rxy \rightarrow Rxfx) \]
\[ \forall x \exists y Rxy \]
\[ \exists x Rxx \]

\[ \exists x \exists y \exists z (Rxy \land Ryz \land Rzx) \]

Exercise 2

Determine whether the following set of sentences is consistent and, if yes, describe a model of the set:

A:  
\[ \exists x \forall y (Pz \land (Ry \rightarrow Qxy)) \]
\[ \forall x \forall y (\neg Px \lor \neg Sy \lor \neg Qxy) \]
\[ \exists y (Ry \land Sy) \]
Exercise 3

Consider the following set of sentences

A: \( \forall x \forall y \forall z. f(xyz) = f(xyfz) \)
\( \exists x \forall y \exists z (fyx = y \land fyz = x) \)

For each of the following three sentences, decide whether it is a consequence of A and, if not, give a model in which A holds, but the sentence fails.

\( F_1: \forall x \forall y \forall z (f(xz) = f(yz) \rightarrow x = y) \)
\( F_2: \forall x \forall y (f(xy) = f(yx)) \)
\( F_3: \forall y \forall y \forall z (f(xz) = f(zy) \rightarrow x = y) \)