EXERCISES FOR METAMATH I

1. Exercise

Find all solutions to the equation $(vu)^n = u^n v^n$ in non-empty words u, v and $n \geq 1$.

2. Exercise

If
$$\alpha = w_1 w_2 w_3 \ldots = w_{k+1} w_{k+2} w_{k+3} \ldots$$
for $w_i \in \Sigma^+$ and $k \geq 1$, show that $\alpha = (w_1 \ldots w_k)^\omega$.

3. Exercise

Let $x, y, z \in \Sigma^+$ and suppose that $xyz = zyx$. Show that there are words $u, v \in \Sigma^+$ and $i, j, k \geq 0$ such that $x = (uv)^i u$, $y = (vu)^j v$ and $z = (uv)^k v$.

4. Exercise

An infinite word α is *recurrent* if any finite subword that occurs in α actually occurs infinitely often.

(a) Show that α is recurrent if and only if any finite subword that occurs in α occurs at least twice.

Given a finite word $w = a_1 a_2 \ldots a_n$, define its *reversal* $\breve{w}$ by $\breve{w} = a_n \ldots a_2 a_1$. An infinite word α is *mirror invariant* if for any word w occurring in α, also $\breve{w}$ occurs in α.

(b) Show that any mirror invariant infinite word α is recurrent.