2013 ATKIN MEMORIAL LECTURE AND WORKSHOP: ABSTRACTS OF TALKS

MAY 3–5, UIC

1. “Combinatorial stability and representation stability” by Thomas Church (Stanford)

Abstract: If you choose a squarefree polynomial $f(T) \in \mathbb{F}_q[T]$ uniformly at random, it will have slightly less than one linear factor on average. The exact value of this expectation depends on the degree of $f(T)$, but as $\deg f(T) \to \infty$ the expectation stabilizes and converges to $1 - \frac{1}{q} + \frac{1}{q^2} - \frac{1}{q^3} + \cdots$.

I will explain how the stabilization of this combinatorial formula, and other statistics like it, is equivalent to a representation-theoretic stability in the cohomology of braid groups. I will describe how combinatorial stability for statistics of squarefree polynomials, of tori in $\mathrm{GL}_n(\mathbb{F}_q)$, and other geometric counting problems can be converted to questions of representation stability in topology, and vice versa. (Joint work with J. Ellenberg and B. Farb.)

2. “Arithmetic statistics, monodromy, and connected components” by Jordan Ellenberg (UW Madison)

Abstract: When you transpose the Cohen-Lenstra problem to the function field setting, you are counting points on certain Hurwitz spaces over finite fields; what’s more, these Hurwitz spaces are always covers of a fixed space (in this case configuration space) whose monodromy groups are (larger and larger) symplectic groups over a fixed finite field. Thus, the connected components of the relevant spaces are given by orbits of symplectic groups in certain linear actions, and counting these orbits is closely related to counting points on the spaces themselves.

3. “Analogues of Cohen-Lenstra for elliptic curves” by Wei Ho (Columbia)

Abstract: I will discuss methods to obtain average sizes of Selmer

groups for certain families of elliptic curves. One somewhat surprising application is to ranks of elliptic curves over quadratic extensions. (This is joint work with M. Bhargava.)

4. " Stability in the homology of congruence subgroups" by Andy Putman (Rice).

Abstract: I will discuss representation-theoretic patterns in the homology of congruence subgroups of $SL_n(\mathbb{Z})$.

5. "TBA" by Sam Ruth (Princeton).

Abstract: TBA

6. "The secondary term in counting cubic fields." by Arul Shankar (IAS).

Abstract: The Davenport–Heilbronn theorem provides the main term in the asymptotics for the number of cubic fields having bounded discriminant. We establish a second main term for these theorems, thus proving a conjecture of Roberts. This is joint work with Manjul Bhargava and Jacob Tsimerman.

7. "Explicit Dirichlet Series for Fields With Given Resolvent" by Frank Thorne (South Carolina).

Abstract: Building on work of H. Cohen and his collaborators, we will describe a Kummer-theoretic approach to enumerating number fields. This leads to explicit formulas for the Dirichlet series counting cubic (or, more generally, degree l) fields with given quadratic resolvent, and for the series counting A_4 or S_4 fields with given resolvent. This is joint work with H. Cohen.

This was motivated by my previous work on Shintani zeta functions, and I will sketch how it follows that the Shintani zeta function can't be written as a finite sum of Euler products. The present work is also related to Ohno and Nakagawa's "extra functional equation" for the Shintani zeta function, and I will describe how I hope our work might lead to extensions of Ohno and Nakagawa's work.

8. "The secondary term in counting quartic fields" by Jacob Tsimerman (Harvard).

Abstract: The secondary term in counting quartic fields Abstract: We discuss secondary main terms for quartic fields having bounded discriminant. Our main tool is a 'slicing' method, extending Davenports lemma for counting lattice points to certain thin sets. This is joint work with Arul Shankar.

9. Atkin Memorial Lecture "Cohen-Lenstra heuristics" by Akshay Venkatesh (Stanford).

Abstract: I will discuss some models of what a "random abelian group" is, and some conjectures (the Cohen-Lenstra heuristics of the title) about how they show up in number theory. I'll then discuss the function field setting and a proof of these heuristics, with Ellenberg and Westerland. The proof is an example of a link between analytic number theory and certain classes of results in algebraic topology ("homological stability").

10. "Bhargava's heuristics for large degree" by Akshay Venkatesh (Stanford).

Abstract: Bhargava has formulated heuristics about the number of S_n fields of fixed degree and bounded discriminant. I'll discuss a couple of examples of what happens when we apply these heuristics out of warranty, e.g. fixed discriminant and large degree.

11. "Homology of Hurwitz spaces and the Cohen-Lenstra heuristics." by Craig Westerland (Melbourne).

Abstract: We will examine Hurwitz moduli spaces of branched covers of Riemann surfaces, and study their stable homology (as the number of branch points tends to infinity). The results may be used to prove a function field analogue of the Cohen-Lenstra heuristics on the distribution of class groups of imaginary quadratic number fields. The main tools are a homological stability theorem for these spaces, as well as an identification of their limit as a space of functions into a target which supports a certain "universal" branched cover.

12. "Abelian and non-abelian Cohen-Lenstra moments" by Melanie Matchett Wood (UW Madison).

Abstract: The Cohen-Lenstra heuristics for the class groups of real quadratic fields are partly based on an analogy with function fields, where the affine class group is Pic^0 modulo a certain divisor, and an assumption that Pic^0 follows the imaginary quadratic heuristics and the divisor is random. We prove the moments of the distribution agree with the latter more refined assumptions for q sufficiently large depending on the moment. We also will introduce the non-abelian Cohen-Lenstra heuristics due to Boston, Bush, and Hajir, and give results on their moments in the imaginary quadratic (joint with Boston and Ross) and real quadratic cases. We will explain what the Boston-Bush-Hajir heuristics predict the moments should be, and the cases in which we can prove the moments in the function field case.