From Apostol page 18

THEOREM I.5. \(a(b-c) = ab-ac\)

PROOF:

By THEOREM I.2, \(ab-ac\) is the unique number \(x\) such that \(ac + x = ab\)

To show the given equality we will show that \(a(b-c)\) also solves that equation:

\[
ac + a(b-c) = ac + a(b+(-c)) \quad \text{by THEOREM I.3}
\]
\[
= ac + ab + a(-c) \quad \text{by distributive axiom}
\]
\[
= ac + a(-c) + ab \quad \text{by commutative axiom}
\]
\[
= a(c+(-c)) + ab \quad \text{by distributive axiom}
\]
\[
= a \cdot 0 + ab \quad \text{by definition of negative as given in THEOREM I.2}
\]
\[
= a \cdot (0+b) \quad \text{by distributive axiom}
\]
\[
= ab \quad \text{by existence of identity elements}
\]

But, \(ab-ac\) is the only number \(x\) such that \(ac + x = ab\). So \(a(b-c) = ab-ac\).