EXAMPLES (5 pts each)
Provide an example with a brief explanation or prove an example does not exist.

1. An accumulation point that is not in the boundary of a set.

2. A function, $\mathbb{R} \rightarrow \mathbb{R}$, that is not continuous on the integers, but is continuous at all other numbers.

3. A function that is 1-1 but not onto.

4. An intersection of open sets that is not open.

5. A bounded sequence that does not converge.

6. A sequence that converges to the real number 0.3.
TRUE OR FALSE (5 POINTS EACH)
Circle TRUE or FALSE.
Provide a counterexample or a brief explanation.
Provide a rigorous proof of one of the true statements.

1. TRUE OR FALSE  Every convergent sequence is bounded.
2. TRUE OR FALSE  There exists a $y$ such that for all $x, y > x^2$.
3. TRUE OR FALSE  For all $x$ there exists a $y$ such that $y > x^2$.
4. TRUE OR FALSE  The quotient of two continuous functions is continuous.
5. TRUE OR FALSE  A point, $p$, is an accumulation point of $S$ if and only if every ball $B$ about $p$ intersects $S$.
6. TRUE OR FALSE  If $T \subseteq S$, then the int $T \subseteq$ int $S$.
7. TRUE OR FALSE  A boundary point is either an accumulation point or an isolated point.
PROVE ONE OF THE FOLLOWING, (35 POINTS)
Correct statements of the definitions should be included in the proofs.
Circle the statement that you are proving.

1. Using the open set definition of continuous, prove that the composition of two functions that are both continuous from $\mathbb{R} \rightarrow \mathbb{R}$ is continuous.

2. Show that, if $f: \mathbb{R} \rightarrow \mathbb{R}$ is continuous by the open set definition then $f$ is continuous by the $\varepsilon$-$\delta$ definition.

3. Using the $\varepsilon$-definition of convergent sequences, prove:

   if $a_n \rightarrow a$ and $b_n \rightarrow b$, then $a_n \cdot b_n \rightarrow a \cdot b$