Complete this make-up exam and bring it to the review session on Monday December 8.

NAME:____________________________________

EXAMPLES (5 pts each)
Provide an example with a brief explanation or prove an example does not exist.

1. A boundary point that is not an accumulation point.

2. A function, $\mathbb{R} \rightarrow \mathbb{R}$, that is not continuous at every point.

3. A union of closed sets that is not closed.

4. A set in which every point is boundary point.

5. A bounded sequence that does not have a convergent subsequence.

6. A sequence that converges to the real number 0.9.

Make-up five more examples like these. Provide justification. Include at least one for which there is no example.
TRUE OR FALSE (5 POINTS EACH)
Circle TRUE or FALSE.
Provide a counterexample or a brief explanation.
Provide a rigorous proof of one of the true statements.

1. TRUE OR FALSE  Every bounded sequence converges.
2. TRUE OR FALSE  The empty set and $\mathbb{R}$ are the only two subsets of $\mathbb{R}$ that are both open and closed.
3. TRUE OR FALSE  If the product of two functions is continuous, then both of the functions are continuous.
4. TRUE OR FALSE  A closed set contains all of its accumulation points.
5. For each one, circle $\mathbf{T}$, TRUE OR $\mathbf{F}$, FALSE.

6. TRUE OR FALSE  An accumulation point is either an interior point or a boundary point.

Make up 6 more TRUE or FALSE questions. Include at least two FALSE statements and at least two TRUE statements. Provide answers and explanations.
PROVE ONE OF THE FOLLOWING. (20 POINTS)
Prove rigorously and directly from the definitions used in Morgan. Correct statements of the definitions should be included in the proofs. Circle the statement that you are proving.

1. A finite set has no accumulations points.

2. If $f$ is continuous from $\mathbb{R} \to \mathbb{R}$, use the $\varepsilon$-$\delta$ definition of continuous to show that if $a_n \to a$, then $f(a_n) \to f(a)$.

3. Using the $\varepsilon$-definition of convergent sequences, prove:

   if $a_n \to 0$ and $b_n \to b$, then $a_n \cdot b_n \to 0$

If $f$ and $g$ are continuous from $\mathbb{R} \to \mathbb{R}$, use the $\varepsilon$-$\delta$ definition of continuous to show that $f \cdot g$ is also continuous.