What’s Wrong with this Proof? Make comments on what needs to be improved and correct errors in the proof. As much as possible you want to say what’s wrong here and how to correct it rather than to write your own proof.

Theorem I.13 \( (a/b) + (c/d) = (ad + bc)/(bd) \) if \( b \neq 0 \) and \( d \neq 0 \)

PROOF:
we know \( ad + bc = ad + bc \)

so \( 1/bd (ad + bc) = (ad + bc) 1/bd \) By I.8 – start here. It’s true by Ax. Also consider adding parentheses to avoid confusion. By conventional rules you don’t mean what is written.

\( (ad)/(bd) + (bc)/(bd) = (ad + bc)/(bd) \) By Axiom 3 – there is a missing step you are using I.9, that is \( A \cdot B^{-1}=(A/B) \). The step should be written out. You use it differently on each side of the equation.

\( (a/b) + (c/d) = (ad + bc)/(bd) \) -- you are using \( (ad)/(bd)=a/b \). This has not yet been proven. It will follow as a special case of this theorem (when \( c=0 \)). Or you can prove it separately by considering that \( bd(a/b)=bd(ab^{-1})=ad\cdot b^{-1}=ad \). It follows, by uniqueness of division, that \( a/b=ad/(bd) \).

Consider an alternate method that is easier to follow in that it doesn’t involve follow two sides of an equation at once. I’ve left the justifications for you to provide.

\[
\frac{ad + bc}{bd} = (ad + bc)(bd)^{-1}
= (ad)(bd)^{-1} + (bc)(bd)^{-1}
= \frac{ad}{bd} + \frac{bc}{bd}
= \frac{a}{b} + \frac{c}{d}
\]