Week 5. More functions

Review:
Workshop Geometry: Unit 1, Lesson 1 -- the definition of a circle
Workshop Geometry: Unit 2, Lesson 4 -- the Pythagorean theorem

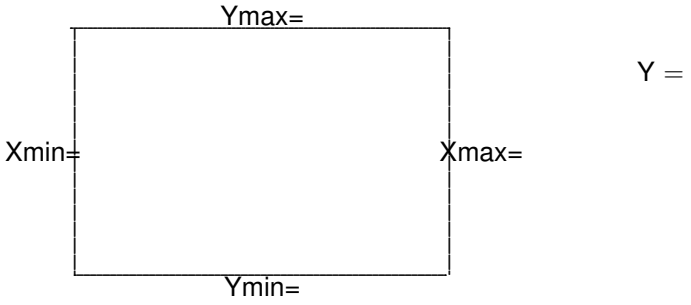
Problem of the Week: The closest point
Find the point on the line $y = 3x + 2$ that is closest to the point $(2, 3)$. (This does NOT mean the closest point that has integer coordinates -- you will need to consider other points.)

Distance functions and circles

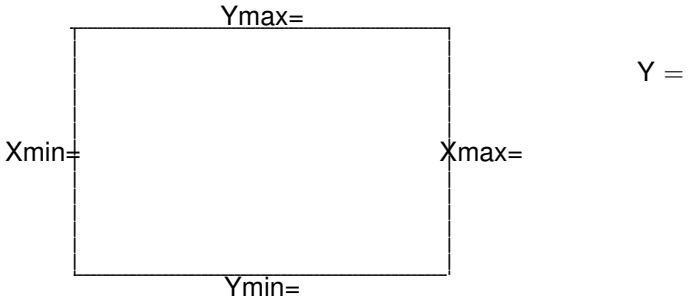
Definition:
The expression $|x - y|$ (we say "the absolute value of $x - y$") means the distance, on a number line, between the two numbers x and y.

91. Explain why the expression $|x|$ is equal to x if x is positive; is equal to $-x$ if x is negative; and is equal to 0 if x is 0.

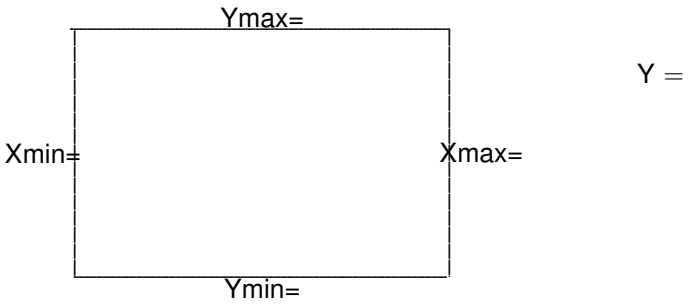
92. Graph the function $y = |x|$ on your calculator. Explain the results.



93. Graph the function $y = |x - 2|$ on your calculator. Explain the results.



94. Graph the function $y = |x + 7|$ on your calculator. Explain the results.



95. Suppose you are driving a constant speed, 60mph, from Chicago to Detroit, about 275 miles away. When you are 120 miles from Chicago you pass through Kalamazoo, Michigan. Sketch a graph of your distance from Kalamazoo as a function of time.

96. Use the Pythagorean Theorem to find the distance between the two points $(1, 3)$ and $(-1, 5)$.

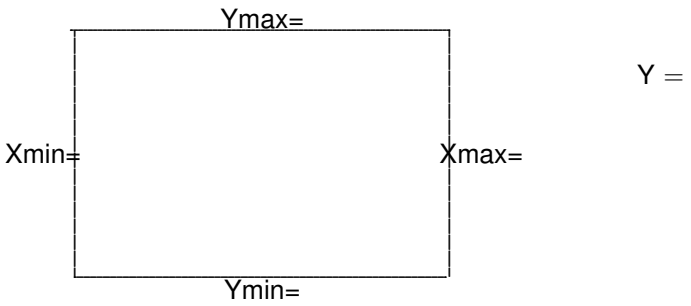
97. Use the Pythagorean Theorem to find a formula for the distance between the two points (c, d) and (a, b).

Definition:
A circle is the set of all points that are equidistance from a fixed point. That fixed point is called the center of the circle. The common distance of a point on the circle from the center is called the radius of the circle.

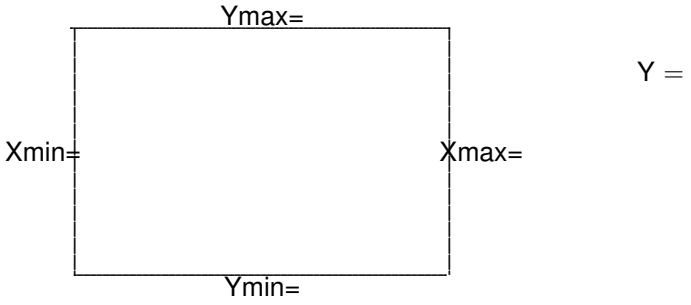
98. Use the Pythagorean Theorem to find an equation for the circle that has a center at $(2, 3)$ and a radius of 2.

You can draw a circle on your graphing calculator in two ways. Using functions, you must enter two functions in order to get both the top and the bottom halves of the circle.

99. On your calculator: draw the circle centered at $(4, 5)$ with a radius 5.



100. On your calculator: draw the graph $(x-2)^2+(y+3)^2=4$. What is the center of this circle? What is the radius of the circle?

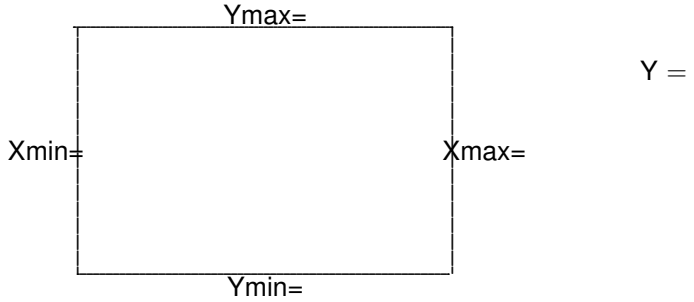


101. On a piece of graph paper, sketch the graph of these circles. Use a compass to make a good circle. Label the $x-$ and $y-$ intercepts. Graph the equation on the calculator to check your sketch.

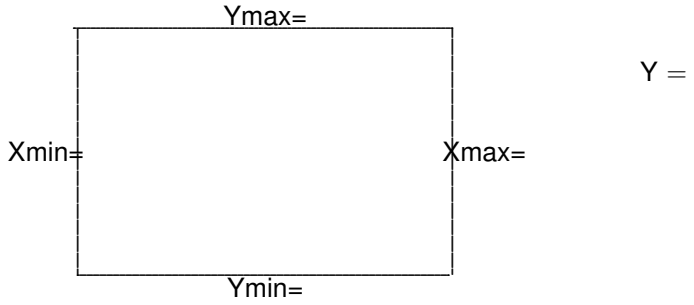
a) $(x+6)^2+y^2=4$ b) $(x-5)^2+(y+2)^2=5$

102. Do these problems on a sheet of graph paper. First sketch the circle, then find an equation for the circle. Check your answer by graphing the circle on your calculator.

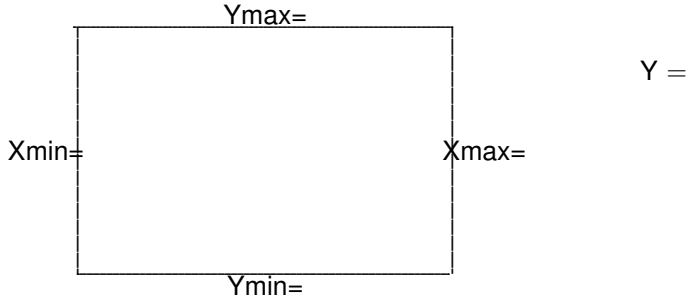
a) Center $(2, 2)$; passes through the origin.



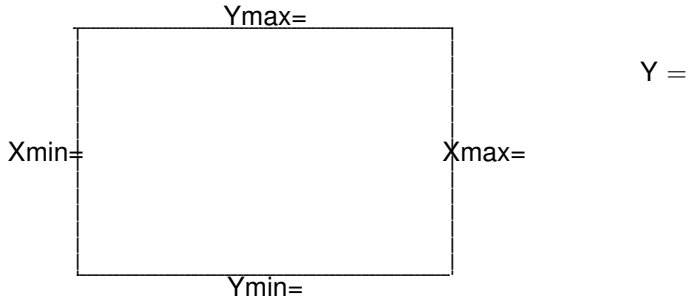
b) Center $(-1, -3)$; passes through $(-4, -2)$.



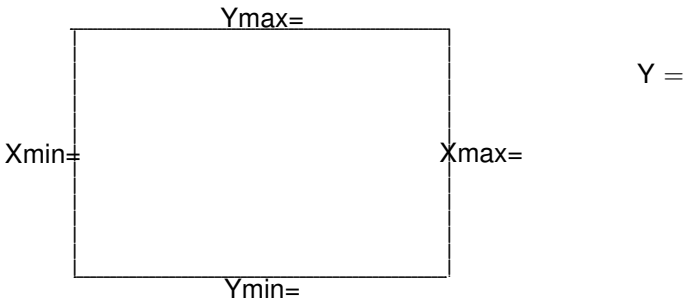
c) Center $(1, 2)$; intersects $x-$axis at -1 and 3.



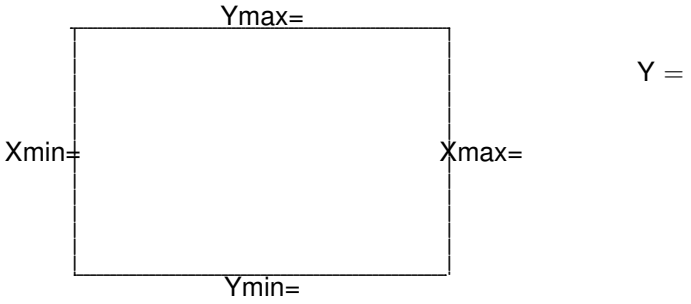
d) Center $(3, 1)$; diameter 2



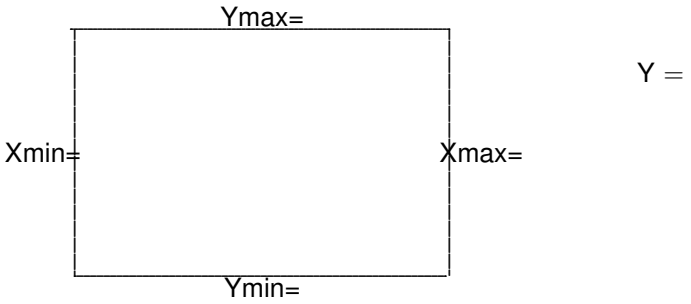
e) Center $(-5, 4)$; intersects the $x-$axis in exactly one point.



f) Center $(2, -6)$; intersects the $y-$axis in exactly one point.



g) Endpoints of diameter are $(3, 3)$ and $(1, -1)$



103. Find the point on the circle $x^2 + y^2 = 25$ that is closest to the point $(1, 3)$.

Exponential functions

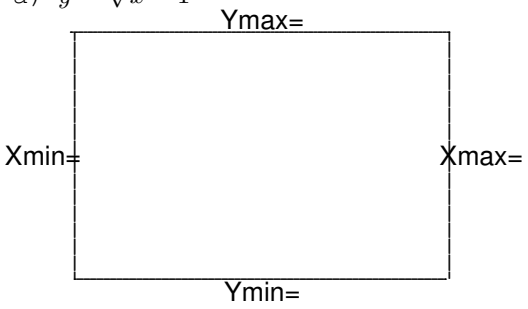
Handout

Hughes-Hallett: Section 1.2

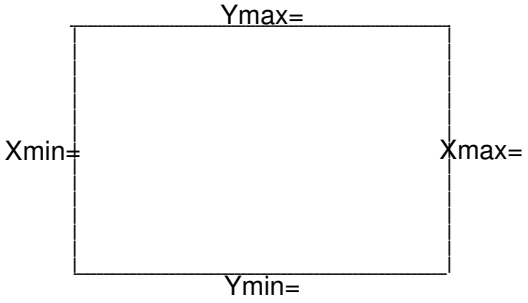
Other functions

104. For each of these functions, first make a table of values, then use your calculator to graph the function. From the graph check the (x, y) pairs in your table. Finally, for each function figure out for what values of x the expression is a real number.

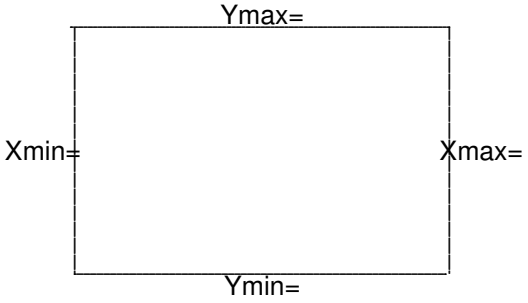
a) $y = \sqrt{x-1}$



b) $y = (x-1)^3$



c) $y = \sqrt{x^2-9}$



105. Functions may be created in many ways. With your calculator, you can explore many different functions just by typing a legitimate calculator expression using the x variable into the function page (Y). Try something interesting. View the graph with different windows and generate tables to review.