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Calculus for Middle School Teachers

Problems and Notes for MTHT 466

Bonnie Saunders Fall 2010

I. Infinity

Week 1. How big is Infinity?

Problem of the Week: The Chess Board Problem

There once was a humble servant who was also a chess master. He taught his king to play the game of chess. The king became fascinated by the game and offered the servant gold or jewels payment, but the servant replied that he only wanted rice: one grain for the first square of chess board, two on the second, four on the third, and so on with each square receiving twice as much as the previous square. The king quickly agreed. How much rice does the king owe the chess master? Suppose it was your job to pick up the rice. What might you use to collect it? A grocery sack, a wheelbarrow, or perhaps a Mac truck? Where might you store the rice?

1. What is the largest number your calculator will display?

2. What is the largest integer your calculator will display that has a 9 in the one's place? What happens if you add 1 to this number?

3. What is the largest number you can think of? Write it down, then write down a larger one.

4. (TEACHING PROJECT) Writing big numbers is made simpler by the use of exponents. Discuss definitions and basic rules for working with positive exponents.

5. The following are common errors made by middle school students when dealing with exponents. Identify the error. Why do you think the student made the error? Write a "teacher" explanation, using the basic rules of exponents, to help the students see the correct use of exponents.

   a) $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 99999$ b) $(5^2)^3 = 5^8$

   c) $3^2 \cdot 3^3 = 9^5$ d) $(2+3)^2 = 4+9$

Sets of numbers

$\mathbf{N} = \{1, 2, 3, 4, 5, ...\}$, Natural numbers.

$\mathbf{W} = \{0, 1, 2, 3, 4, 5, ...\}$, Whole numbers

$\mathbf{I} = \{... -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...\}$, Integers

$\mathbf{Q} = \{\frac{m}{n} : m \in \mathbf{I}, n \in \mathbf{I}, n \neq 0\}$, Rational numbers

$-$ Numbers that can be expressed as a ratio of two integers.

$\mathbf{R}$, Real numbers

6. Something to think about: Are there more @ signs or more \$ signs? How do you tell without counting?

```
@   @   @   @   @   @   @   @   @   @   @   @   @   @   @   @   @   @   @   @   @
$   $   $   $   $   $   $   $   $   $   $   $   $   $   $   $   $   $
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Definition of a one-to-one correspondence

A one-to-one correspondence between sets A and B is a pairing of each object in A with one and only one object in B.

Counting

Finite sets that can be put into a one-to-one correspondence necessarily have the same number of elements. A set that contains n objects can be put into a one-to-one correspondence with the set that contains the first n natural numbers. The process of making this particular one-to-one correspondence is called *counting*. When a child understands one to one correspondence, the child is ready to count.

7. Give examples of pairs of sets that can be put into one-to-one correspondence. Give examples of pairs of sets that can NOT be put into one-to-one correspondence.

8. Give an example of a pair of sets and two different ways that the two sets can be put into one-to-one correspondence.

Infinite sets

Sets with an infinite number of elements can be put into a one-to-one correspondence by using patterns or formulas.

9. Give an example of a pair of sets that have an infinite number of elements that can be put into one-to-one correspondence.

10. In what sense can we say that there are the same number of even whole numbers as there are whole numbers? In what sense can we say that there are twice as many whole numbers as even whole numbers?

11. In some sense, there is one more whole number than there are natural numbers. Explain the sense in which **W** and **N** contain the same number of numbers, that is show a one-to-one correspondence between these two sets.

Cardinality

If there is any way to put two sets into a one-to-one correspondence, we say that they have the same *cardinality*. We might say that the two sets have the same number of elements, but that is imprecise unless the set is finite.

12. Show a one-one-correspondence between **N** and each of the following sets. Show a pattern by making a table and use a formula.

a) the set of odd positive integers

b) the set of negative integers

c) the set of positive integers that are divisible by 3.

d) the set of powers of 10, that is $\{1, 10, 100, 1000, \cdots\}$

13. Show a one-one-correspondence between $\mathbf{W}$ and each of the sets in problem 12.

a) the set of odd positive integers

b) the set of negative integers

c) the set of positive integers that are divisible by 3.

d) the set of powers of 10, that is $\{1, 10, 100, 1000, \cdots\}$

14. CLASS DISCUSSION Are there more rational numbers than whole numbers? Discuss.

Handout

Dodge Ball

15. Write a short paragraph explaining the strategies for Player One and Player Two in the game of Dodge Ball.

16. CLASS DISCUSSION Are there more real numbers than rational numbers?

Sequences

A sequence is a list of numbers. They are often denoted with subscripts,

$$a_0, a_1, a_2, a_3, \cdots, a_n, \cdots$$

Sequences continue forever so that there is a one-to-one correspondence with W. The subscripts can also start with 1, instead of 0. (Or, in fact, any other integer, positive or negative) The subscript numbers are called *term numbers*. In elementary school, sequences are written down in tables. In middle school, children begin to learn how to write formulas for the numbers in the tables.

17. Joe has 107 baseball cards. Every week he buys 5 more cards. How many cards does he have after 3 weeks? How many cards does he have after one year? Make a table to help see the pattern. Write a formula to compute how many cards he has after any number of weeks. Explain your formula.

Arithmetic Sequences: constant consecutive difference

An *arithmetic sequence* is a sequence in which the difference between any two consecutive numbers in the sequence is constant. Arithmetic sequences are important in high school mathematics because formulas for arithmetic sequences correspond to equations for straight lines. The difference is that equations for straight lines use real numbers, not just integers, for the independent variable.

18. Write down three or four examples of arithmetic sequences. Write down three or four sequences that are not arithmetic sequences. Use tables or lists and formulas.

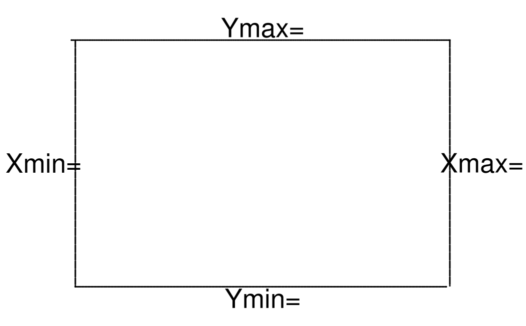
19. Remember an example of an arithmetic sequence from MATH 140 or MATH 141 that was constructed using pictures or objects. Make a table and write a formula for the sequence.

Using the calculator to make sequences

There are several ways to make sequences on a graphing caculator. One common way is to make a function on the $Y =$ screen and then to make a table.

20. Enter a function, Y_1, in your calculator and use it to make the table you created in problem 17.

21. Enter a function, Y_1, in your calculator and use it to make a table containing the arithmetic sequence $3, 10, 17, 24, 31, \ldots$. Graph the function with a reasonable window and record it below. Include the values for the window.



Y =

PRACTICE WITH LISTS

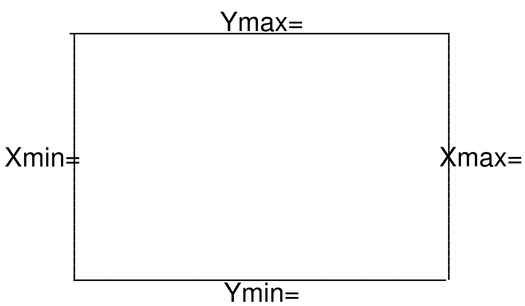
22. Make a list in your calculator containing the arithmetic sequence:

$$a_0 = 5, a_1 = 2, a_2 = -1, a_3 = -4, ...$$

What is the 1000^{th} term in this sequence? What is a_{1000}? Why are they not the same?

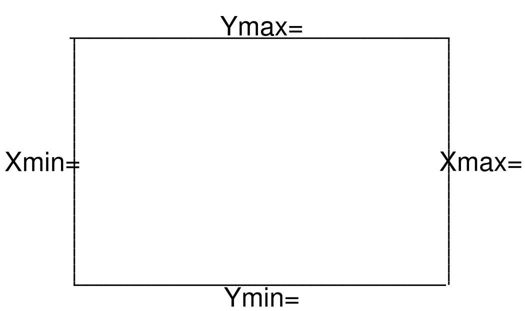
23. Make a list in your calculator containing each of these arithmetic sequences. Graph your results and answer the question.

a) $0.5, 0.75, 1.0, 1.25, ...$
If $0.5 = a_0$, what is the term number for 10.75?



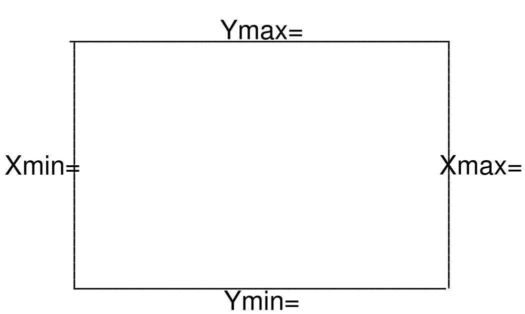
Y =

b) $0, 18, 36, 54, ...$
What number in this sequence is closest in value to 10000?



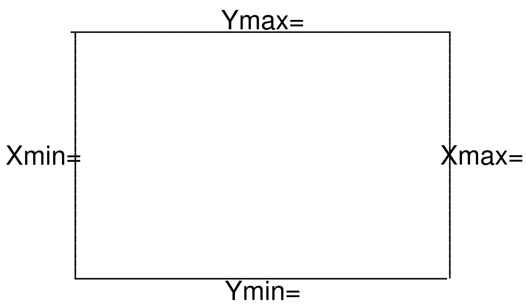
Y =

c) $159, 148, 137, 126, ...$
What is the first negative number in this sequence?



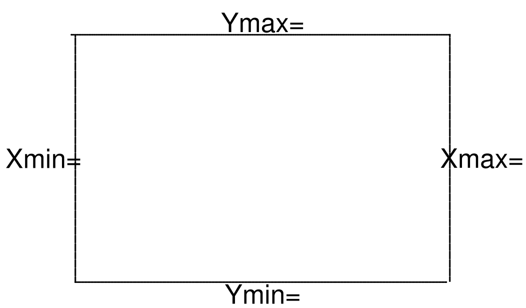
Y =

d) Make up an arithmetic sequence such that the 95th term is 0. Show the graph.



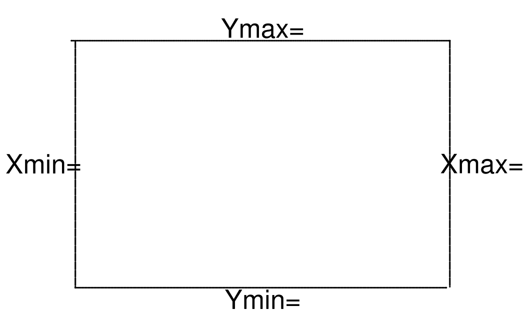
Y =

e) Make up an arithmetic sequence such that the first term is 23 and the 15th term is 51. Do you think that everyone will make up the same sequence?



Y =

f) Make up an arithmetic sequence such that the fifth term is 40 and the 9th term is 12. Do you think that everyone will make up the same sequence?



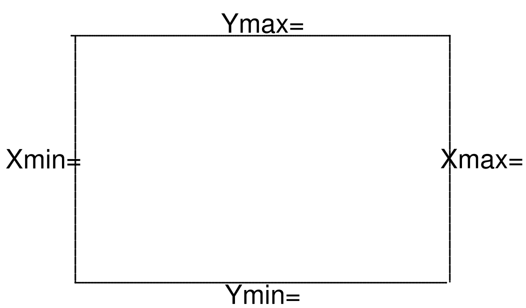
Y =

24. Consider the Problem of the Week and make a table to see the pattern. Write a formula to compute how many grains of sand there are on the nth square of the checkerboard. Explain your formula.

Geometric Sequences: constant consecutive ratios

A geometric sequence is a sequence in which the ratio between any two consecutive numbers is always the same. The sequence you made in problem 24 is a geometric sequence. The constant ratio is 2.

25. Enter a function, Y_1, in your calculator and use it to make a table containing the geometric sequence $1, 2, 4, 8, 16, \ldots$? Graph the function with a reasonable window and record it below. Include the values for the window.

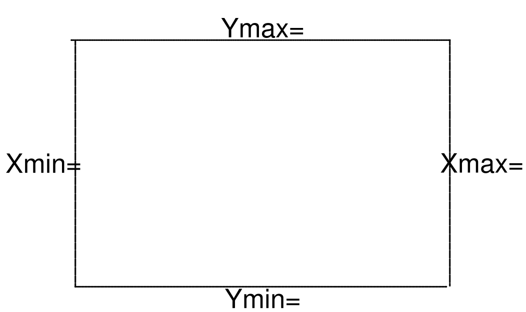


Y =

26. Consider the sequence that is the reciprocals of the numbers in the sequence in problem 25:

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$$

Graph it.



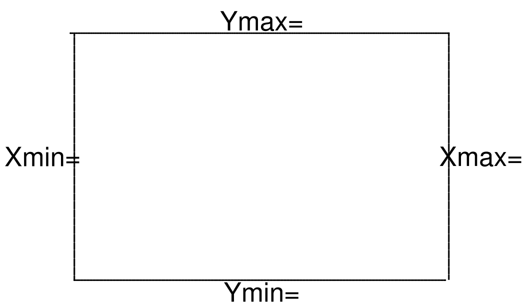
Y =

What is the smallest number in this sequence that your calculator can display exactly? What is the exact decimal expansion for the next number in the sequence?

MORE PRACTICE WITH LISTS

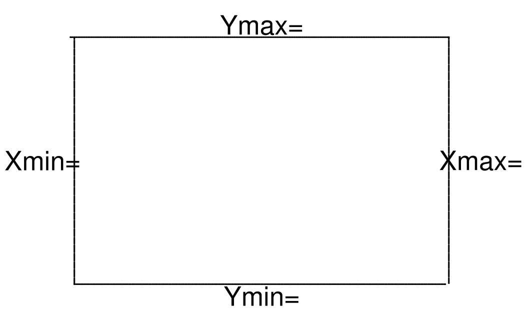
27. Make a list in your calculator containing each of these geometric sequences. Make a function and a table for each one. Plot and graph your results and answer the question.

a) $0.3, 0.09, 0.027, .0081, ...$
What number in this sequence is closest in value to 10^{-10}?



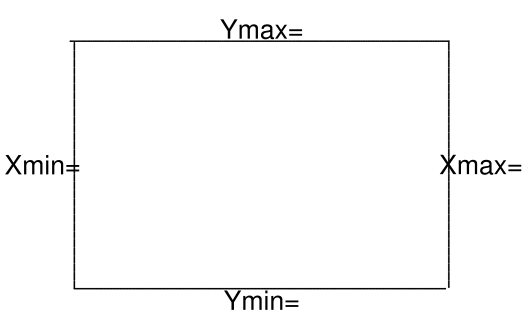
Y =

b) $6, 18, 54, 162, ...$
What number in this sequence is closest in value to 10000?



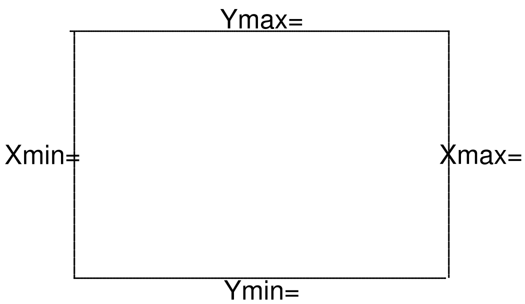
Y =

c) $15000, 3750, 937.5, ...$
What is the first number in this sequence that is less than 1?



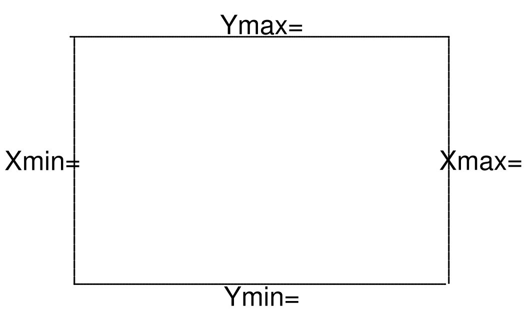
Y =

d) Make up a geometric sequence such that the 15th term is 1.
Do you think that everyone will make up the same sequence?



Y =

e) Make up an geometric sequence such that the third term is 3 and the 7th term is 48.
Do you think that everyone will make up the same sequence?



Y =

Other Kind of Sequences

28. Decide on a likely pattern that continues each of these sequences. Make a table on your calculator for each of these sequences. Display the 500th term of each on your calculator and determine whether that value is exact or approximate (or too big or too small for your calculator to determine)?

a) $1, 4, 9, 16, \ldots$

Ymax=

Xmin= Xmax=

Ymin=

Y =

b) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

Ymax=

Xmin= Xmax=

Ymin=

Y =

c) $0, 2, 6, 12, 20, \ldots$

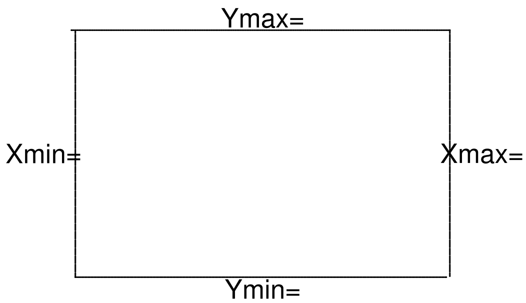
Ymax=

Xmin= Xmax=

Ymin=

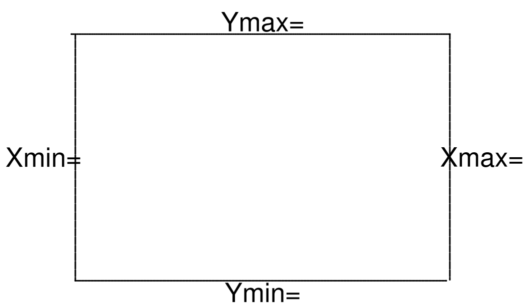
Y =

d) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$



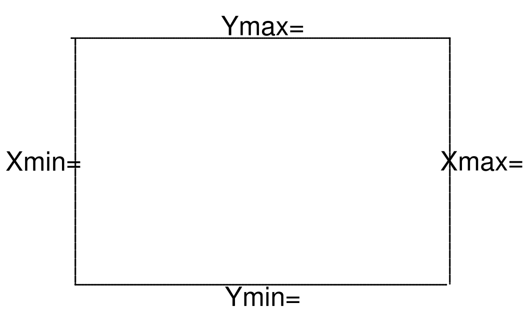
Y =

e) $1.0, 0.1, 0.01, 0.001, 0.0001, \ldots$



Y =

f) $1.0, 0.89, 0.78, 0.67, \ldots$



Y =

29. Conjecture and prove a formula for the nth term of an arithmetic sequence. Carefully describe all the variables in your formula.

30. Conjecture and prove a formula for the nth term of a geometric sequence. Carefully describe all the variables in your formula.

Week 2. How small is infinity? – Limits

Problem of the Week: A probability investigation

Suppose we have a box containing 5 white balls, 3 black balls, and 2 red balls. What is the probability that a ball, drawn at random, will be black? One at a time we add 3 balls – one of each color – to the box and recompute the probability of drawing a black ball. Is it possible that the probability will ever be $\frac{1}{3}$? What is the largest possible probability that can be realized?

31. What is the smallest positive number your calculator will display?

32. What is the number closest to $\frac{1}{3}$ that your calculator will display? Is this number – the one displayed – greater than or less than $\frac{1}{3}$? What is the difference between the two numbers? Write down a number that is closer to $\frac{1}{3}$ than the one your calculator displays.

33. Repeat all parts of problem 32 for $\frac{2}{3}$.

34. What is the number closest to $\sqrt{2}$ that your calculator will display? Can you tell if this number – the one displayed – is greater than or less than $\sqrt{2}$?

35. Give an several examples of what is suggested by these rather imprecise statements:

$$\frac{1}{BIG} = LITTLE$$

$$\frac{1}{LITTLE} = BIG$$

$$\frac{1}{CLOSETO1} = CLOSETO1$$

36. (TEACHING PROJECT) Writing small numbers is made simpler by the use of negative exponents. Discuss definitions and basic rules for working with non-positive exponents. Research: Find a reference for these definitions and rules from a book or on the internet.

37. The following are common errors made by middle school students when dealing with negative exponents. Identify the error. Why do you think the student made the error? Write a "teacher" explanation, in terms of the definition of exponents, to help the students see the correct use of exponents.

a) $\frac{1}{10^{-3}} < 1$

b) $10^{\frac{1}{4}} = \frac{1}{10^4}$

c) $0.0002356 \cdot 10^{20} < 1$

d) $7.459817 \cdot 10^{10}$ is not an integer

e) $28911 \cdot 10^{-100} > 1$

Definition: $\lim_{n\to\infty} a_n = 0$

No matter how small a positive number you can think of, there is a value of n large enough so that $\frac{1}{n}$ is smaller than that number for every single value of n that is greater than N.

38. Explain: For any integers, n and N, $n \geq N$ if and only if $\dfrac{1}{n} \leq \dfrac{1}{N}$

39. Consider: $\lim_{n\to\infty} \dfrac{1}{n} = 0$. Think of a very smaller number and call it e. Find an integer N such that $\dfrac{1}{N} < e$? Think of an even smaller value for e and find N such that $\dfrac{1}{N} < e$. Identify a procedure for finding such an N given any e.

Rules for working with limits

The sub-sequence rule for working with limits

If $\lim_{n\to\infty} a_n = 0$, then the limit of any subsequence is also 0.

The squeeze rule for working with limits

If $0 \leq b_n \leq a_n$ for all values of n and if $\lim_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty} b_n = 0$

The multiplication rule for working with limits

If $\lim_{n\to\infty} a_n = 0$ and c is some number, then $\lim_{n\to\infty} c \cdot a_n = 0$.

40. Which rule tells you that the limit is equal to 0?

a) $\lim_{n\to\infty} 10^{-n}$

b) The sequence $0.3, 0.03, 0.003, 0.0003, 0.00003, \ldots$.

c) $\lim_{n\to\infty} \frac{1}{\sqrt{n^3}}$

41. Find three more examples of sequences that converge to 0.

Zooming in

Beckmann activities: 2D, 2H, 2J

42. Are there any real numbers contained in all of the intervals, $[-\frac{1}{n}, \frac{1}{n}]$, for all values of n?

43. Draw "zoom-in" picture for each of these sequences of nested interval problems. In each case the nested intervals contain exactly one number. What is that number? Give the value as a fraction of whole numbers, not a decimal.

    a) $[.3, .4], [.33, .34], [.333, .334], \ldots$

    b) The middle third of the middle third of the middle third ... of $[0, 1]$

    c) The first tenth of the last tenth of $[0, 1]$,
    then the last tenth of that interval,
    then the first tenth of that interval,
    then the last tenth of that interval, ...

Sequences and limits

Definition: limits that are not zero

If $\lim_{n\to\infty} a_n - a = 0$, then we say that $\lim_{n\to\infty} a_n = a$ for any positive integer m.

44. Find an example for each of the following rules.

Rules for working with limits

If $\lim_{n\to\infty} a_{n+m} = a$ for some positive integer m, then $\lim_{n\to\infty} a_n = a$

For any real number b, if $\lim_{n\to\infty} a_n = a$, then $\lim_{n\to\infty} a_n + b = a + b$

For any real number c, if $\lim_{n\to\infty} a_n = a$, then $\lim_{n\to\infty} ca_n = ca$

If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then $\lim_{n\to\infty} a_n + b_n = a + b$

If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then $\lim_{n\to\infty} a_n b_n = ab$

If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$ *and* if $b \neq 0$, then $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{a}{b}$

If $a_n < b_n$ for all values of n, then $\lim_{n\to\infty} a_n \leq \lim_{n\to\infty} b_n$

The Advanced Squeeze Rule for working with limits

If $a_n \leq b_n \leq c_n$ for all values of n and if $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = d$, then $\lim_{n\to\infty} b_n = d$

A sequence may no have a limit

There are two ways by which a sequence may not have a limit.

1. The terms in the sequence may increase (or decrease) beyond all bounds. In this case, we would write $\lim_{n\to\infty} a_n = \infty$ (or $\lim_{n\to\infty} a_n = -\infty$). Examples: $1, 2, 3, 4, \ldots$ (or $-2, -4, -8, -16, \ldots$)
2. Two different subsequences may have different limits. Example: $1.9, 4.99, 1.999, 4.9999, 1.99999, 4.999999 \ldots$

45. Give examples of sequences that do not have limits.

46. Identify a pattern for each of the following sequences. Identify the limit exactly and explain how you know it is the limit. If there is no limit, explain why it doesn't exist.

a) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

b) $1.0, 0.1, 0.01, 0.001, \ldots$

c) $6.1, 6.01, 6.001, 6.0001, 6.00001, \ldots$

d) $5.9, 5.99, 5.999, 5.9999, \ldots$

e) $0.3, 0.33, 0.333, 0.3333, 0.33333, \ldots$

f) $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots$

g) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$

h) $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots$

i) $1.1, 1.9, 1.01, 1.99, 1.001, 1.999, \ldots$

j) $2\frac{1}{2}, 3\frac{1}{3}, 4\frac{1}{4}, 5\frac{1}{5}, \ldots$

k) $1.8, 1.88, 1.888, 1.8888, \ldots$

47. USING A CALCULATOR TO GUESS A LIMIT. Find a pattern and use your calculator to guess whether or not there is a limit for each of these sequences and what the limit may be. You should find more values of the sequence than those that appear on this page.

a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, \ldots$

b) $\sqrt{\frac{1}{2}}, \sqrt{\sqrt{\frac{1}{2}}}, \sqrt{\sqrt{\sqrt{\frac{1}{2}}}}, \ldots$

c) $\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots$

d) $a_n =$ the fractional part of $(.375n)$

e) $4\sqrt{2-\sqrt{2}}, 8\sqrt{2-\sqrt{2+\sqrt{2}}}, 16\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}, \ldots$

f) $2^1, (\frac{3}{2})^2, (\frac{4}{3})^3, (\frac{5}{4})^4, \ldots$

g) $2\sin(90°), 3\sin(60°), 4\sin(45°), 5\sin(36°), \ldots$

h) $4\sin(90°), 8\sin(45°), 16\sin(22.5°), \ldots$

i) $4\tan(90°), 8\tan(45°), 16\tan(22.5°), \ldots$

Week 3. Adding an infinity of numbers

Problem of the Week: A nickel vs million

It's your first day on a new job and you have your choice of two ways to be paid:
METHOD #1: You get one nickel the first day. From then on everyday you get one and one half times what you got the day before.
METHOD #2: You get a million dollars every day.
Which pay schedule would you choose and why?

First some formulas for adding finite sums

Handout

Picture proofs

48. Confirm each of these formulas for $n = 1$ to $n = 20$ by making a table of values:

a) $1 + 2 + 3 + 4 + 5 + \ldots + n = \frac{1}{2}n(n + 1)$ or $\sum_{k=1}^{n} k = \frac{1}{2}n(n + 1)$

b) $1 + 2^2 + 3^2 + 4^2 + 5^2 + \ldots + n^2 = \frac{1}{6}n(n + 1)(2n - 1)$ or $\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n + 1)(2n - 1)$

c) $1 + 3 + 5 + \ldots + 2n - 1 = n^2$ or $\sum_{k=1}^{n} (2k - 1) = n^2$

49. Confirm each of this statements, guess a general formula, write it with summation notation. Can you think of a picture proof for your formula?

$$1 + \frac{1}{2} = 2 - \frac{1}{2}$$

$$1 + \frac{1}{2} + \frac{1}{4} = 2 - \frac{1}{4}$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 2 - \frac{1}{8}$$

50. Confirm each of this statements, guess a general formula, write it with summation notation. Can you think of a picture proof for your formula?

$$1 + 2 = 2^2 - 1$$

$$1 + 2 + 4 = 2^3 - 1$$

$$1 + 2 + 4 + 8 = 2^3 - 1$$

Geometric Series

Handout

Hughes-Hallett: Section: 9.1: #15, #16, #17, #18

51. Draw a picture to show how to add an infinite number of numbers.

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots+\frac{1}{2^n}+\ldots=\sum_{n=1}^{\infty}\frac{1}{2^n}$$

52. Use base ten blocks to show that $0.\overline{3}=\frac{1}{3}$. How is this infinite series involved in your argument?

$$\frac{3}{10}+\frac{3}{10^2}+\frac{3}{10^3}+\frac{3}{10^4}+\ldots+\frac{3}{10^n}+\ldots=\sum_{n=1}^{\infty}\frac{3}{10^n}$$

53. Give a convincing argument that $0.\overline{9}=1$. How is this infinite series involved in your argument?

$$\frac{9}{10}+\frac{9}{10^2}+\frac{9}{10^3}+\frac{9}{10^4}+\ldots+\frac{9}{10^n}+\ldots=\sum_{n=1}^{\infty}\frac{9}{10^n}$$

54. Show that for any real number, $x \neq 1$,

$$\sum_{k=0}^{N} x^k = 1 + x + x^2 + x^3 + \ldots + x^N = \frac{x^{N+1} - 1}{x - 1}$$

Summing an infinite number of numbers

We can compute the sum an infinite number of terms by taking limits, for example we say that

$$\sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \ldots + x^N + \ldots = \lim_{N \to \infty} \sum_{k=0}^{N} x^k$$

Looking at the result from 54 we can see that

$$\sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \ldots + x^N + \ldots = \lim_{N \to \infty} \frac{x^{N+1} - 1}{x - 1}$$

Now we just take the limit as $N \to \infty$. If $0 \leq x < 1$, then $\lim_{N \to \infty} x^N = 0$, so, in the end, the limit on the right is $\frac{1}{1-x}$ and we can have

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \ldots + x^n + \ldots = \frac{1}{1-x}$$

You will see another formula that has another variable. This formula comes about by multiplying everything by a constant, r.

$$\sum_{n=0}^{\infty} rx^n = r + rx + rx^2 + rx^3 + rx^4 + \ldots + rx^n + \ldots = \frac{r}{1-x}$$

You can use these two formulas, or modify them slightly to do the next problem.

55. Write each series using summation notation and find the limit. In each case the left hand problem gives you a formula to use for the right hand problem.

a) $1 + x + x^2 + x^3 + \ldots + x^n + \ldots =$

$1 + .1 + .01 + .001 + .0001 + \ldots + .1^n + \ldots =$

b) $x^3 + x^4 + \ldots + x^n + \ldots =$

$\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots =$

c) $1 + x^2 + x^4 + \ldots + x^{2n} + \ldots =$

$1 + \frac{1}{9} + \frac{1}{81} + \frac{1}{729} + \ldots =$

d) $x^3 + x^6 + x^9 + x^{12} + \ldots =$

$0.001 + 0.000001 + 0.000000001 + \ldots =$

56. Find the limit for

$$\frac{21}{100} + \frac{21}{10000} + \frac{21}{1000000} + \cdots + \frac{21}{10^{2n}} + \cdots$$

Explain why your answer is the fraction form for the repeating decimal $0.\overline{21}$.

57. Find the fraction form for each of these repeating decimals.

a) $0.\overline{4}$

b) $0.\overline{36}$

c) $3.\overline{21}$

d) $5.1\overline{21}$

e) $3.4\overline{21}$

f) $51.3\overline{14}$

Using the calculator to compute other infinite sums

58. Other infinite sums may or may not have limits like the geometric series does. Use your calculator (or other means) to conjecture whether or not any of these infinite sums are finite.

a) $1 + 2 + 3 + 4 + 5 + \ldots + n + \ldots =$

b) $1 + 1 + 1 + 1 + 1 + \ldots + 1 + \ldots =$

c) $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{n(n+1)} \ldots =$

d) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n} + \ldots =$

e) $1 + \frac{2}{2} + \frac{3}{4} + \frac{4}{8} + \ldots + \frac{n}{2^n} + \ldots =$

f) $\frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \ldots + \frac{1}{n^2 - 1} + \ldots =$