

Week 6. Transformations in a coordinate system

Review:

Workshop Geometry: Unit 1, lesson 2: Pay particular attention to the parts about using grid paper. Come to class with plenty of graph paper

Problem of the Week: Architectural Arches

See handout, Discovery Project 4

Translations and three Reflections

Whenever possible, do these problems in three ways: making a table, using a graph and modifying a formula. Use your calculator liberally.

106. Start with the circle $\{(x, y) : x^2 + y^2 = 9\}$. Translate the circle so that the center is at $(3, 4)$. Write an equation for the translated circle.
107. Start with the graph of the absolute value function, $\{(x, y) : y = |x|\}$. Translate the graph 4 units to the right. What is the formula for the translated function? Can you give a geometric interpretation of this new function in terms of distances? Make a table of values for both the original and the translated function.
108. Start with the graph, $\{(x, y) : y = |x - 2|\}$. Translate the graph 4 units to the left. What is the formula for the translated function? Make a table of values for both the original and the translated function.
109. Start with the function, $\{(x, y) : y = |x|\}$. Translate the graph so that the vertex is at $(a, 0)$. What is the equation of the new function? What is the difference in the graph if a is positive or negative?
110. Start with the function, $\{(x, y) : y = |x|\}$. Translate the graph so that the vertex is at $(0, b)$. What is the equation of the new function? What is the difference in the graph if b is positive or negative?
111. Start with the parabola, $\{(x, y) : y = x^2\}$. Translate the graph 3 units to the right. What is the formula for the translated function? Make a table of values for both. Explain how the table of values shows the translation.
112. Start with the parabola, $\{(x, y) : y = x^2\}$. Translate the graph 2 units to the left. What is the formula for the translated function? Make a table of values for both. Explain how the table of values shows the translation.
113. Start with the parabola, $\{(x, y) : y = x^2\}$. Translate the graph 3 units to down. What is the formula for the translated function? Make a table of values for both. Explain how the table of values shows the translation.

114. Start with the parabola, $\{(x, y) : y = x^2\}$. Translate the graph 3 units to up and 2 units to the left. What is the formula for the translated function? Make a table of values for both. Explain how the table of values shows the translation.
115. Start with the parabola, $\{(x, y) : y = x^2\}$. Translate the graph so the vertex of the parabola is now at $(2, -4)$. What is the formula for the translated function? Make a table of values for both. Explain how the table of values shows the translation.
116. Start with the parabola, $\{(x, y) : y = x^2\}$. Translate the graph so that the vertex is at (h, k) . What is the equation of the new parabola?
117. Use problem 116 to find the equation of a parabola that has its vertex at the point $(-1, 10)$.
118. Find the number c such that the vertex of the parabola $y = x^2 + 8x + c$ lies on the x -axis.
119. Find a formula for a function whose graph is the reflection about the x -axis of $\{(x, y) : y = x^2\}$. Make a table of values for both. Explain how the table of values shows the reflection.
120. By reflecting and then translating the graph of $\{(x, y) : y = x^2\}$. Find the formula for a parabola that turns down and has a vertex at $(1, 4)$.
121. In the previous problem, what happens if you first translate and then reflect. Do you get the same result?
122. Find a formula for a function whose graph is a parabola that turns up and has a vertex at the point (k, h) .
123. Find a formula for a function whose graph is the reflection about the y -axis of $\{(x, y) : y = x^3\}$.
124. Find a formula for a function whose graph is the reflection about the y -axis of $\{(x, y) : y = \sqrt{x-1}\}$.
125. A function whose graph is symmetric about the y -axis is called an "even" function. Give two examples of even functions.

Stretching and Enlargements

126. The graph of a parabola is obtained from the graph of $y = x^2$ by vertically stretching away from the x -axis by a factor of 2. What is the equation for this parabola?
127. The graph of a parabola is obtained from the graph of $y = x^2$ by vertically shrinking towards the x -axis by a factor of 2. What is the equation for this parabola?

128. For parabolas, explain why stretching in the y -direction (away from the x -axis) looks like shrinking in the x -direction (towards the y -axis).
129. Start with a circle, $x^2 + y^2 = 1$, and stretch it by a factor of 3 in the x -direction. What is the equation of the resulting ellipse?
130. Start with a circle, $x^2 + y^2 = 1$, and stretch it by a factor of 3 in the x -direction and by a factor of 2 in the y -direction. What is the equation of the resulting ellipse?
131. Start with $f(x) = x^2 + 2$, then write the rule of a function whose graph is the graph of f but shifted 5 units to the left and 4 units up.
132. Start with $f(x) = x^2 + 2$, then write the rule of a function whose graph is the graph of f but first shrunk by a factor of 2 towards the y -axis and then shifted 5 units to the left and 4 units up. do you get the same resulting function if you shrink after the translations?

Reflections through $y = x$

133. Start with the line $y = 2x$. Reflect the graph through the line $y = x$. Write a function to graph the reflected line. Repeat for the following lines item $y = 3x$ item $y = 2x + 3$. item $y = -x + 1$. item $y = 5x - 2$. Make a conjecture about the slope of the reflected line?
134. Start with the parabola $y = x^2$. Reflect the graph through the line $y = x$. Can you write a formula for the function this reflected graph?
135. Explain the graph of the function, $f(x) = \sqrt{x-1}$, in terms of a reflection and a translation.
136. Start with the parabola $y = x^2 + 5$. Reflect the graph through the line $y = x$. Can you write a function to graph this reflection? Can you write two functions that will graph this reflection?
137. Start with the parabola $y = -2x^2 + 1$. Reflect the graph through the line $y = x$. Can you write a function to graph this reflection? Can you write two functions that will graph this reflection?
138. Explain the graph of the function, $f(x) = 1 + \sqrt{x-3}$, in terms of translations and a reflection.