IV. Derivatives

Week 11. Tangent lines

Problem of the Week: atop Willis Tower
How far can you see from the SkyDeck viewing area(1353 feet high) of the Sears Tower? If you were allowed to stand on the top of the Sears Tower (1454 feet), how much further could you see?

Review: From Week 4

178. Confirm that the equation of the line that goes through the point, (a,b) and has slope m can be written as $y = m(x-a)+b$.

179. Find conditions on the coefficients, a, b, and c, so that the equation, $ax^2+bx+c=0$, has exactly one solution.

Tangent line to circles

180. Find the best explanation for this fact: The tangent line to a circle is perpendicular to a radius of the circle.

For the following, sketch the circles and the tangent lines before computing. Afterwards, graph the function and the tangent line on the calculator to confirm your answers.

181. Find equations for the lines that are tangent to the circle, $x^2+y^2 = 1$ at the points $(0,1)$, $(1,0)$, $(-1,0)$, and $(0,-1)$.

182. Find an equation for the line that is tangent to the circle, $x^2+y^2 = 25$ at the point $(3,4)$.

183. Find an equation for the line that is tangent to the circle, $x^2 + y^2 = 4$ at the point $(\sqrt{2}, -\sqrt{2})$.

184. Find an equation for the line that is tangent to the circle, $(x+1)^2 + (y-1)^2 = 25$ at the point $(2, 5)$.

optional
Tangent lines from a point to a circle

185. When is a triangle inscribed in a circle a right triangle?

186. Find all the tangent lines to the circle, $x^2 + y^2 = 1$, that go through the point $(1, 1)$.

187. Find all the tangent lines to the circle, $x^2 + y^2 = 1$, that go through the point $(0, 4)$.

Tangent line to parabolas

For the following, use the fact that the tangent line intersects a parabola in only one point to get algebraic equations to solve.

188. Find the equation of the tangent line to the curve at the given point. Graph both the function and the tangent line to confirm your answer.

a) $y = x^2$ at $(1, 1)$.

b) $y = x^2$ at $(-2, 4)$.

c) $y = 3x^2 - 2x + 1$ at $(0, 1)$.

189. Find the equation of the tangent line to the curve at the given point. Graph both the function and the tangent line to confirm your answer. Use transformations to make it easy

a) $y = x^2 + 2x + 1$ at $(0, 1)$.

b) $y = 3x^2$ at $(1, 3)$.

c) $y = x^2 + 1$ at $(2, 5)$.

190. Find the equation of the tangent line to the curve at the given point. Graph both the function and the tangent line to confirm your answer. Use transformations to make it easy

a) $y = x^2 + 2x$ at $(0, 0)$.

b) $y = 3x^2 + 5$ at $(1, 8)$.

c) $y = \sqrt{x}$ at $(1, 1)$.

Derivative at a point.

Definition: the derivative at a point.
For a curve, $y = f(x)$, we name the slope of the tangent line at a point $(a, f(a))$, $f'(a)$, and call it the derivative of f at the point a.

191. Find a curve that does not have a tangent line at some point.

192. Show that $f'(a) = 2a$ if $f(x) = x^2$ for all values of a.