## Week 9. Parabolas and Area

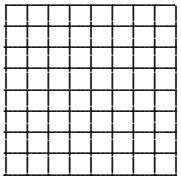
### Review: From Week 1

Formula for the sum of squares. Show that

$$\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n-1)$$

### Problem of the Week: How Many Squares?

There are 64 1x1 squares in this checkerboard. How many squares of all different sizes can you find in this picture?



## The area bounded, in part, by a parabola.

154. Make a guess: what is the area bounded by $y = x^2$, the $x$-axis and the line $x = 1$.

155. Approximate the area bounded by $y = x^2$, the $x$-axis and the line $x = 1$ using four subdivisions of the interval $[0, 1]$ on the $x$-axis.

156. Approximate the area bounded by $y = x^2$, the $x$-axis and the line $x = 1$ using five subdivisions of the interval $[0, 1]$ on the $x$-axis.

157. Approximate the area bounded by $y = x^2$, the $x$-axis and the line $x = 1$ using six subdivisions of the interval $[0, 1]$ on the $x$-axis.

158. Find an expression for the area bounded by $y = x^2$, the $x$-axis and the line $x = 1$ using $n$ subdivisions of the interval $[0, 1]$ on the $x$-axis.

159. Find the area bounded by $y = x^2$, the $x$-axis and the line $x = 1$. In other words, compute $\int_0^1 x^2 dx$.

Draw a picture and then find the area.

160. What is the area under $f(x) = x$ on the interval [0,1]? In other words, compute $\int_0^1 x dx$

161. What is the area under $f(x) = 1 - x^2$ on the interval [0,1]? In other words, compute $\int_0^1 (1 - x^2) dx$

162. What is the area under $f(x) = 1 - x^2$ on the interval [-1,1]? In other words, compute $\int_{-1}^1 (1 - x^2) dx$

163. What is the area under $f(x) = x^2 + 3$ on the interval [0,1]? In other words, compute $\int_0^1 (x^2 + 3) dx$

164. Do you think it is true that $\int_0^1 3x^2 dx = 3 \int_0^1 x^2 dx$? Explain your thinking

165. Compute $\int_1^3 4x + 1 dx$

166. Compute $\int_{-1}^1 |x| dx$

167. Compute $\int_{-2}^0 (x^2 + 2x + 1) dx$

### Changing scale in the x direction

168. What is the area under $f(x) = x^2$ on the interval [0,4]? In other words, compute $\int_0^4 x^2 dx$

169. Compute $\int_0^2 x^2 dx$

170. What is the area under $f(x) = x^2$ on the interval [0.5,1]? In other words, compute $\int_{0.5}^1 x^2 dx$

171. Find a formula for $\int_0^x t^2 dt$.

#### Rules for integrals

If $A \leq C \leq B$, then $\int_A^B f(x)dx = \int_A^C f(x)dx + \int_C^B f(x)dx$

$$\int_A^B af(x) + bg(x)dx = a\int_A^B f(x)dx + b\int_A^B g(x)dx$$

$$\int_{cA}^{cB} f(cx)dx = \frac{1}{c}\int_A^B f(x)dx$$

Use these rules to do the next few problems.

172. Evaluate $\int_0^1 (x^2 + x)dx$.

173. Find a formula for $\int_0^1 (ax^2 + bx + c)dx$.

174. Find a formula for $\int_A^B (x^2 + 3x + 5)dx$.

175. Using the formulas you developed, compute $\int_1^2 1 - x^2 dx$. Explain why the answer is negative.

176. Carefully graph the following functions shade the area(s) indicated by the integral and compute the integral:

a) $\int_{-1}^{0} (2x^2 + 3x + 2)dx$.

b) $\int_{0}^{4} (x^2 - 4x + 3)dx$.

c) $\int_{-1}^{1} (3x^2 - 2x + 1)dx$.

## Integrals that are NOT areas

Hughes-Hallett: Section 5.1

## Definition Integral

Hughes-Hallett: Section 5.2 – Computing using the definition

177. Write a program to approximate integrals on your calculator. Test the program on many integrals.