Identifying Identities

An **identity for real numbers** is an equation involving mathematical expressions that holds true when any real numbers are substituted for the variables. For example, $a + b = b + a$, is an identity that because commutative rule of addition holds for all real numbers.

On the other hand, $a + b = a$, is not an identity because it is only true if $b = 0$. And $\sqrt{a^2} = a$ is not an identity because it is not true if any negative number is substituted for the variable a.

Which of the following are identities? For each identity, give a convincing argument that the equation is always true. For each equation that is not an identity, give examples when the equation is false.

1. $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$

2. $(a+b)^2 = a^2 + b^2$

3. $(2ab)^2 = 4(ab)^2$

4. $2(a \cdot b) = (2a \cdot 2b)$

5. $3 \cdot (a+b) = 3 \cdot a + 3 \cdot b$

6. $(a+b)^2 = a^2 + 2a \cdot b + b^2$

7. $4(a \cdot b) = (2a \cdot 2b)$

8. $\sqrt{a^2 + 2a \cdot b + b^2} = a + b$

9. $(a+b) \cdot (c+d) = ac + ad + bc + bd$