From http://www.transum.org/Software/sw/Starter_of_the_day/. First, fill in the blanks. Then figure out how many in a gross and how many in a score.

A dozen, a gross, and a score,
Plus three times the square root of _ _ _ _
Divided by _ _ _ _ _
_ _ _ _ five times eleven
Equals _ _ _ _ squared and not a bit more.

1. Manny has four pennies, four nickels and four quarters. What amounts of money can he make exactly from these coins? [For example, he can make 53¢ by taking 3 pennies and 2 quarters] Explain.

2.

a. How many different ways can you make 79¢ using just pennies, nickels and quarters? Justify your answer in some way.

b. What is the smallest number of coins that can be used to make 88¢? Explain your reasoning. Describe a procedure for finding the smallest number of coins to make any amount of money.

3. Pencils are often packaged by the dozen (12 pencils to a package) and boxed by the gross (12 packages or 144 pencils). Explain how to think of this method of counting as using a number system with a different base. Include examples in your explanation.

4. How many three digit numbers are there in base 6? How many in hexadecimal? How many in the decimal? What is the pattern?

5. Explain, through examples, algorithms for doing addition, multiplication, and division using base six arithmetic.

A puddle is a closed interval of numbers. For example [3, 7] contains all the numbers 3, 4, 5, 6, and 7.

1.

a. If a +7 cricket starts on -9 write a formula for where he lands on his nth jump.

b. Where might a -53 cricket start to land in the puddle [377, 381]? List all possible starting points.

2.

a. Does a +27 cricket, starting at zero, land in any of these puddles?

[1011, 1038] [5771, 5766] [14390, 14392]

b. Give a convincing argument that a +47 cricket will land in the puddle [12788, 12834] as long as it starts on a number less than 12788.

3. The boxes indicate the numbers a certain positive cricket jumped on. Each box is one jump. What kind of cricket is it? Fill in the rest of the boxes.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | 6 | | | | 82 | | |

Try another one:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | -113 | | | | | | 19 | |

Explain a procedure for doing this kind of problem.

4. If a -37 cricket starts at 1,000,000, after how many jumps will it land in [10378, 10418]? Will it land in [972, 975]?

5. Joe has 107 baseball cards. Every week he buys 5 more cards. How many cards does he have after 3 weeks? How many cards does he have after one year? What does this problem have to do with crickets?

6. On a new block in town there are fifteen brand new houses in a row. The first house on the block is to be numbered 1101 and the last house is to be numbered 1465. What numbers should the other houses have if the numbers are to be evenly spaced? What does this problem have to do with crickets? Explain a way to do this problem without drawing a picture.

7. Joe's annual income has been increasing each year by the same amount. The first year his income was $44,000 and the ninth year his income was $59,840. In which year was his income first more than $51,500? Rewrite this problem as a cricket problem.

8. The Excellent Health club offers its members two options:
   i. A one-time initial fee of $100 plus a membership fee of $40 per month.
   ii. A one-time initial fee of $300 plus a monthly membership fee of $30.
   Discuss the relative merits of each plan. Rewrite this problem as a cricket problem.

9. A +4 cricket starts at 139. At the same time a -6 cricket starts at 1042. If they both jump at the same time every time, will they ever land on the same number at the same time? Draw a picture of the relevant part of the number line showing how the two crickets cross paths.

10. If a +4 Cricket starts at 3 will it land on any powers of 10? Which ones? If a +3 Cricket starts at 4 will it land on any powers of 10? Which ones?

Whenever possible include a combination chart with your explanation of these problems.

1. In a football game, a touch down with an extra point is worth 7 points and a field goal is worth 3 points. Suppose that in a game the only scoring done by teams are touchdowns with extra points and field goals. Which of the scores from 1 to 25 are impossible for a team to score? List all ways for a team to score 40 points.

2. I only have 5¢ and 13¢ stamps. What postages can I make, given a sufficient numbers of each kind of stamp? What is the largest postage that I cannot make? For example, I obviously cannot make 6¢.

3. A customer wants to mail a package. The postal clerk determines the cost of the package to be $2.86, but only 6¢ and 15¢ stamps are available. Can the available stamps be used for the exact amount of the postage? Why or why not?

4. Joe counts 48 heads and 134 legs among the chickens and dogs on his farm. How many dogs and how many chickens does he have?

5. A cow is worth 10 pieces of gold, a pig is worth 5 pieces of gold and a hen is worth 1 piece of gold. 220 gold pieces are used to buy a total of 100 cows, pigs and hens. How many of each animal is bought?

6. Terry has some 5 oz. weights, some 7 oz weights and a two-pan balance. Show how she can weigh out 1 ounce, 2 ounces, 3 ounces, 4 ounces, 11 ounces and 12 ounces chocolates. For example, she could put a 5 oz. weight on one side of the balance and a 7 oz weight on the other side; then she could add chocolate to the 5-oz side until it balanced the other side. Is there any weight she could not weigh out given that she has sufficient 5-oz and 7-oz weights?

7. **Fountain of Knowledge** (Adapted from The Heart of Mathematics, by Starbird and Burger)
During an incredibly elaborate hazing stunt during pledge week, Trey Sheik suddenly found himself alone in the Sahara desert. His desire to become a fraternity brother was now overshadowed by his desire to find something to drink (these desires, of course, are not unrelated). As he wandered aimlessly through the desert sands, he suddenly came upon an oasis.

There, sitting in a shaded kiosk beside a small pool of mango nectar, was an old man named Al Donte. Big Al ran the mango bar and informed Trey that the juice was sold only in 8-ounce servings and that the cost for one serving was $3.50. Trey searched his pockets for change and discovered that he had exactly $3.50.

Trey's jubilation at the thought of liquid coating his dried and chapped throat was quickly shattered when Al casually announced that there were no 8-ounce glasses available. Al had only a 6-ounce glass and a 10-ounce glass - neither of which had any markings on them. Al, being a man of his word, would not hear of selling any more or any less than an 8-ounce serving of mango juice. Do you think it is possible to use only the unmarked 6- and 10-ounce glasses to produce exactly 8 ounces of juice in the 10-ounce glass? Explain.

Whenever possible include a combination chart with your explanation of these problems.

8.

a. On a 6, 8 Combination Chart will a ↑↑→cricket, starting on a12, ever land on 1027?

b. On the same chart, describe the arrows for a cricket that starts at 12 and next lands on another 12.

c. On a 1, 5 combination chart describe arrows for a +22 cricket. Describe a procedure for finding the arrows for any size cricket.

d. On a 4, 7 combination chart describe the arrows that make a +1 cricket. Is it possible to make a +1 cricket on a 4, 6 combination chart.

e. On a n, m combination chart find arrows that make a +0 cricket.

9. Complete the rest of this section of a combination chart. Make an EXCEL version that includes a space that contains zero.

| | | | | |
|---|---|---|---|---|
| | 181 | | | |
| | | | 172 | |
| | | | | |
| | | | | |
| 109 | | | | |

Show how to solve this problem by solving equations.

10. **Challenge:** I only have 31¢ and 37¢ stamps. What postages can I make, given a sufficient numbers of each kind of stamp? What is the largest postage that I cannot make?

11. You have some 12 oz. weights, some 15 oz weights and a two-pan balance. What are all the other weights can you determine?

**MTHT 467** **Problem Set #4** **Spring 2010**

Include in your discussion of these problems connections to the concepts lcm and/or gcd.

1.

a. The bookstore marked down some notepads from $2.00 but still kept the price over $1.00. It sold all of them. The total amount of money from the sale of the pads was $31.45. How many notepads were sold?

b. A group of people ordered No-Cal candy bars. The bill was $2.09. If the original price of each was 12¢ but the price has been inflated, how much does each cost?

c. The radio station gave away a discount coupon to every fifth caller and a CD to every sixth caller. Every twentieth caller received free concert tickets. Which caller was first to get both a CD and a concert ticket? Which caller was first to get all three prizes? If there were 150 callers, how many of each prize did they give away?

2.

a. Larry and Mary bought a special 360-day joint membership to a tennis cub, Larry will use the club every other day, and Mary will use the club every third day. They both use the club on the first day. How many days will neither person use the club in the 360-days?

b. At a party store, paper plates come in packages of 30, paper cups in packages of 40, and napkins in packages of 75. What is the least number of packages of plates, cups, and napkins that can be purchased so that there is an equal number of each item?

c. Two bells ring at 8:00 am for the remainder of the day, one bell rings every half hour and the other bell rings every 45 min. What time will it be when the bells ring together again?

3.

a. On a string of Christmas tree lights, the red ones blink every 3 seconds, the blue ones blink every 4 seconds and the white blinks every 4.5 seconds. What is the maximum number of times they all blink together in a one-hour interval?

b. Jane and Roman are running laps on a track. If they start at the same time and place and go in the same direction, with Jane running a lap in 5 min and Ramon running a lap in 3 min, how long will it take for them to be at the starting place at the same time if they continue to run at these speeds? How many laps will each of them run?

c. Three runners are running on a circular track. The first completes one lap every 4 minutes. The second completes one lap every 6 minutes, the third every 8 minutes. If they start together, when is the first time they get to the starting line at the same time? At that time, how may laps has each completed?

4.

a. The red line bus takes 20 minutes to complete its route from the time it leaves from and returns to the station. The blue line bus takes 25 minutes to complete its route from the time it leaves from and returns to the station. If both buses begin their routes at 6:00 am how many times throughout the day will they meet at the station at the same time, if the busses stop running at 11:00pm? When is the first time the will meet?

b. Don loves peanut butter and jelly sandwiches. One day while he was eating, he noticed that each jumbo jar of peanut butter has 72 servings, but the jelly jar has only 40 servings. If he opened jars on the same day and used exactly one serving each day, how many days would it take until he emptied a peanut butter jar and a jelly jar on the same day?

c. You have a square pattern with which you would like to tile a room that is 203 feet by 77 feet. You want the square design to be as big as possible and you do not want any gaps or borders, the squares must exactly tessellate the area. What is the largest square pattern you can do?

5. According to D.Wells, the following problem was posed by Sun Tsu Suan-Ching (4th century AD): There are certain things whose number is unknown. Repeatedly divided by 3, the remainder is 2; by 5 the remainder is 3; and by 7 the remainder is 2. What will be the number?

6. Seventeen robbers stole a bag of silver coins. They divided the coins into equal groups of 17 but 3 were left over. A fight began over the remaining coins and one of the robbers was killed. The coins were the redistributed but this time 10 were left over. Another fight broke out and another of the robbers died in the conflict. Luckily, another equal redistribution of the coins was exact. What was the least number of coins stolen by the robbers?

7. A woman with a basket of eggs finds that if she removes the eggs from the basket 3 at a time, there is 1 egg left. If she removes the eggs from the basket 4 at a time, there is 1 egg left. If she removes the eggs from the basket 5 at a time, there is 1 egg left. However, if she removes the eggs 7 at a time, there are no eggs left. If the basket holds no more than 200 eggs, how many eggs are in the basket?

**MTHT 467** **Problem Set #5** **Spring 2010**

1. Leap years occur in years that are divisible by four. However, if the year ends in two zeros, in order for the year to be a leap year, it must be divisible by 400.

   a. Suppose today is Friday. What day of the week will it be 43 days from today? What day of the week will it be 76 days from today? How does division with remainder help solve this problem?

   b. Halloween (October 31) of 2007 was on a Wednesday. Determine what day of the week Halloween will be on in 2010. When is the next year that Halloween will be on a Wednesday?

   c. November 22, 2007 was Thanksgiving. What was the date of Thanksgiving in 2006? What will be the date of Thanksgiving in 2010?

2. Must there be at least one Friday the $13^{th}$ in every year?

3. A certain number,
when divided by 10 has a remainder of 8;
when divided by 9 has a remainder of 7;
when divided by 8 has a remainder of 6;
when divided by 7 has a remainder of 5;
when divided by 6 has a remainder of 4;
when divided by 5 has a remainder of 3;
when divided by 4 has a remainder of 2;
when divided by 3 has a remainder of 1;
when divided by 2 has a remainder of 0;
What is the smallest positive number satisfying these conditions?

4. 112 is the difference between two squares, $11^2 - 3^2$. What other numbers can be made from the difference of two squares? Discuss your method for finding them.

5.

   a. Jimmy Dorsey, the big band leader, was born on leap day 1904 and died on June 12, 1957. How old was he when he died and how many birthdays did he have?

   b. Herman Hollerith, inventor of the electronic tabulating machine, was born on leap day 1860 and died on January 17, 1929. How old was he when he died and how many birthdays did he have?

   c. William Wellman, director of such movies as A Star is Born and The Oxbow Incident, was born on leap day 1896 and died on February 9, 1976. How old was he when he died and how many birthdays did he have?