Problem Set 7
Solutions

1. $X \div 3 = r\,2 \rightarrow$ 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35
$X \div 5 = r\,3 \rightarrow$ 8, 13, 18, 23, 28, 33,
$X \div 7 = r\,2 \rightarrow$ 9, 16, 23

Check

$23 \div 3$: $23 - 21 = 2$, so $7\ r\ 2$

$23 \div 5$: $23 - 20 = 3$, so $4\ r\ 3$

$23 \div 7$: $23 - 21 = 2$, so $3\ r\ 2$

$\boxed{\text{The number is 23}}$

2. A) $X \div 17 = r\,3$
B) $X \div 16 = r\,10$
C) $X \div 15 = r\,0 \rightarrow$ [this means # ends in 5 or 0]

A) 3 mod 17 (ending in 5, or 0)
20, 105, 190, 275, 360, 445, 530, 615, 700, 785, 870, 955, 1040, 1125, 1210, 1295, 1380, 1465, 1550, 1635, 1720, 1805, 1890, 1975, 2060, 2145, 2330, 2315, 2400, 2485, 2570, 2655, 2740, 2825, 2910, 2995, 3080, 3165, 3250, 3335, 3420, 3505, 3590, 3675, 3760, 3845, 3930.

B) 10 mod 16 (ending in 0 or 5)
90, 170, 250, 330, 410, 490, 570, 650, 730, 810, 890, 970, 1050, 1125, 1210, 1290, 1370, 1450, 1530, 1610, 1690, 1770, 1850, 1930, 2010, 2090, 2170, 2250, 2330, 2410, 2490, 2570, 2650, 2730, 2810, 2890, 2970, 3050, 3130, 3210, 3290, 3370, 3450, 3530, 3610, 3690, 3770, 3850, 3930

C) ~~1210 ÷ 15 = 80.6~~
~~2570 ÷ 15 = 171.3~~
$\boxed{3930 \div 15 = 262.0}$

$\boxed{\text{3930 is the least coins}}$

3. multiples of 7:

7 14 21 28 35 42 49 56 63 70 77 84 91 98

- multiples of 7 that also divided by 5 have a remainder of 1: 21, 56, 91

- multiples of 7 that divided by 5 have a remainder of 1 & divided by 3 have a remainder 1: 91

- multiples of 7 that divided by 5 have a remainder of 1 & divided by 3 have a remainder 1 & divided by 4 have a remainder 1: None

$\boxed{\text{Impossible}}$

4\. 100 coins (last $\frac{2}{3}$ left by robbers)
\+ 50 coins (last $\frac{1}{3}$)
  2 coins (hush money)
152 coins

152 coins (last $\frac{2}{3}$ left by 2nd robber)
\+ 76 coins (last $\frac{1}{3}$)
  2 coins (hush money)
230 coins

230 coins (last $\frac{2}{3}$ left by 1st robber)
\+ 115 coins (last $\frac{1}{3}$)
  2 coins (hush money)
347 coins

[illegible]

*Best method is to work backwards

$\boxed{\text{347 coins}}$

Examples

$\frac{10}{10}$

Problem set #7 [redacted]

1) A certain number...
divided by 3 has remainder 2 ① $x \equiv 2 \pmod 3$
divided by 5 has remainder 3 ② $x \equiv 3 \pmod 5$
divided by 7 has remainder 2 ③ $x \equiv 2 \pmod 7$

*good to repeat problem

+2

① $x = 3t + 2$ substitute into ② $x \equiv 3 \pmod 5$

② $3t + 2 \equiv 3 \pmod 5$

$3t \equiv 1 \pmod 5$

$t \equiv 2 \pmod 5$

*very structured

1 mod 5 = 1	6 mod 5 = 1
2 mod 5 = 2	7 mod 5 = 2
3 mod 5 = 3	8 mod 5 = 3
4 mod 5 = 4	9 mod 5 = 4
5 mod 5 = 0	10 mod 5 = 0

$t = 5s + 2$ substitute into ① $x = 3t + 2$

① $x = 3(5s + 2) + 2$

$x = 15s + 6 + 2$

$x = 15s + 8$

*Notice mod format

$x = 15s + 8$ substitute into ③ $x \equiv 2 \pmod 7$

$15s + 8 \equiv 2 \pmod 7$

Casting out 7 gives $s \equiv 1 \pmod 7$

$s = 7u + 1$

$s = 7u + 1$ substitute into $x = 15s + 8$

$x = 15(7u + 1) + 8$

$x = 105u + 15 + 8$

$x = 105u + 23$

1 mod 7 = 1	8 mod 7 = 1
2 mod 7 = 2	9 mod 7 = 2
3 mod 7 = 3	10 mod 7 = 3
4 mod 7 = 4	11 mod 7 = 4
5 mod 7 = 5	12 mod 7 = 5
6 mod 7 = 6	13 mod 7 = 6
7 mod 7 = 0	14 mod 7 = 0

$105 = \text{lcm}(3, 5, 7)$

$\boxed{\text{SOLUTIONS} = 23, 128, 233 \ldots}$ *gave other possible solutions

2) A certain number

4) Let x be the total number of coins, we knew that there are 100 coins left at the end of the story, working backward.

*Explained where each # came from

- 100 is the 2 parts the third robbers left after he gave two coins to the servant and kept one part of the coin.
- Thus, $100 \div 2 = 50$ coins in each part which means $50 \times 3 = 150$ coins the third had before he gave the two coins to servant which increase the number of coins to 152.
- 152 coins is the two parts the second robbers left in the pile, $152 \div 2 = 76$ coins in each part. $(76 \times 3) + 2 = 230$ coins the second robbers had before he gave two coins to the servant,
- Same for the first robbers. $[(230 \div 2) \times 3] + 2 = (115 \times 3) + 2 = 347$.
- Thus, 347 coins were in the pile originally.

*very neat & logical

+2

Proof:

$347 - 2 = 345$ First robber gave two coins to the servants

$345 \div 3 = 115$ keep: 115, left: $(115 \times 2) = 230$

$230 - 2 = 228$ second robber gave two coins to servant.

$228 \div 3 = 76$ keep $= 76$, left $(76 \times 2) = 152$

$152 - 2 = 150$ third robber gave two coins to servant

$150 \div 3 = 50$ keep: 50, left $(50 \times 2) = 100$ ✓

In the morning they found 100 coins left.

*great idea to prove answer!

~~2970, 3210, 3450, 3690~~ $\boxed{3930}$, ~~4170~~.
~~I found that subtract 3 from 3930 can be divisible by 17.~~
+3
~~Test it out.~~
~~$3930 \div 15 = 262$ ✓~~
~~$(3930 - 10) \div 16 = 245$ ✓~~
~~$(3930 - 3) \div 17 = 231$ ✓~~
~~Thus, the least number of coins is 3930 coins.~~

3) We knew that there is less than 100 eggs, and the number of eggs can be divisible by 7. Then, I list out the multiples of 7.
*Repeated problem
7, 14, ~~21~~, ~~28~~, ~~35~~, ~~42~~, 49, ~~56~~, ~~63~~, ~~70~~, 77, ~~84~~, 91, 98.
- I cross out the multiples of 3, 4, and 5, and left with the number 7, 14, 49, 77, 91, 98.
+3
Test each number with the multiples of 5. *Tested #s

$7 - 5 = 2$ ×	$77 - 75 = 2$ ×
$14 - 10 = 4$ ×	$91 - 90 = 1$ ✓
$49 - 45 = 4$ ×	$98 - 95 = 3$ ×

Then, test 91 with the multiples of 3 and 4.
$91 - 90 = 1$ ✓ $\quad$ $91 - 88 = 3$ ×

*Explained by it was impossible instead of just stating it is
Thus, this problem does not have the solution since 91 is the number less than 100, and it matched only three requirments, not four.

Problem Set 7
Solutions

1. $X \div 3 = r\,2 \rightarrow 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35$
$X \div 5 = r\,3 \rightarrow 8, 13, 18, 23, 28, 33,$
$X \div 7 = r\,2 \rightarrow 9, 16, 23$

Check

$$\begin{array}{r} 7\,r2 \\ 3\overline{)23} \\ -21 \\ \hline 2 \end{array} \qquad \begin{array}{r} 4\,r3 \\ 5\overline{)23} \\ -20 \\ \hline 3 \end{array} \qquad \begin{array}{r} 3\,r2 \\ 7\overline{)23} \\ -21 \\ \hline 2 \end{array}$$

The number is 23

2. A) $x \div 17 = r\,3$ *Repeated problem
B) $x \div 16 = r\,10$
C) $x \div 15 = r\,0$ → [this means # ends in 5 or 0]

A) 3 mod 17 (ending in 5, or 0) *started w/ easiest
20, 105, 190, 275, 360, 445, 530, 615, 700, 785, 870, 955, 1040, 1125, 1210, 1295, 1380, 1465, 1550, 1635, 1720, 1805, 1890, 1975, 2060, 2145, 2330, 2315, 2400, 2485, 2570, 2655, 2740, 2825, 2910, 2995, 3080, 3165, 3250, 3335, 3420, 3505, 3590, 3675, 3760, 3845, 3930.

B) 10 mod 16 (ending in 0 or 5) *found patterns
90, 170, 250, 330, 410, 490, 570, 650, 730, 810, 890, 970, 1050, 1125, 1210, 1290, 1370, 1450, 1530, 1610, 1690, 1770, 1850, 1930, 2010, 2090, 2170, 2250, 2330, 2410, 2490, 2570, 2650, 2730, 2810, 2890, 2970, 3050, 3130, 3210, 3290, 3370, 3450, 3530, 3610, 3690, 3770, 3850, 3930

C) ~~1210 ÷ 15 = 80.6~~ *very neat & logical
~~2570 ÷ 15 = 171.3~~
$\boxed{3930 \div 15 = 262.0}$

3930 is the least coins