MTHT 467 **MIDTERM 1** **Spring 2012**

NAME: *ANSWER KEY*

1. Which of the following crickets will ever land in a painty region between 87 and 92? Provide reasons. Try for the most elegant, efficient reason.

A +10 cricket starting at 16
Adding 10 to 16 (& all subsequent #'s) preserves the ones digit, so this cricket lands on every number that ends in a 6, which are not found between 87 and 92.

A +10 cricket starting at -2
-2 + 9(10) = 88 OR
+10 cricket lands on 8 and all subsequent #'s with an 8 in the ones place; this includes 88.

A +8 cricket starting at -8
-8 + 12(8) = 88
-8 is starting point; 12 is the number of jumps; +8 is the cricket.

A +3 cricket starting at 15
15 + 25(3) = 90 OR
Region has length = 92 – 87 = 5, which is larger than a jump of a +3 cricket. Therefore it will always land in the region (as long as you start below 87).

A +6 cricket starts jumping at 3. Later, a +4 cricket starts jumping at 7. Will the +4 cricket ever come along and land on a place where the +6 cricket has already landed?

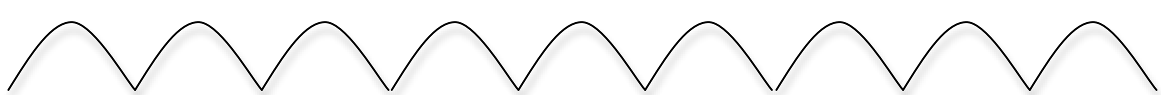
Yes, the first place they both land at is 15 because 3 + 2(6) = 15 = 7 + 2(4).

Write an algebraic expression that describes all such places.

In 3 jumps the +4 cricket advances 12 units—the same # of units the +6 cricket advances in 2 jumps. Mathematically, this (12) is the Least Common Multiple (lcm) of 4 & 6. Since they both start at 15 now, they will meet 12 units later at 27 (the +6 cricket in 2 jumps, and +4 crickets in 3 jumps). That is, 15 + 2(6) = 27 = 15 + 3(4). After 15, they both land on every 12^{th} spot. This is an arithmetic sequence of numbers given by the formula:

$$15 + 12n \text{ where } n \geq 0.$$

2. What kind of +cricket made these jumps? Fill in the rest of the landing numbers.



47 *54* **61** *68* *75* **82** *89* *96* *103* *110*

The distance between 82 and 61 is 82 – 61 = 21. 21 units divided into 3 equal jumps makes each jump 7 units, therefore the cricket is a +7 cricket.

3. Which of the following are identities? If it is an identity, show why using rules of arithmetic. If not an identity, provide a counterexample.

a. $(a+b)^2 = a^2 + b^2$

Let $a = -1$ *and* $b = 1$.

Left Hand Side (LHS): $(a+b)^2 = (-1+1)^2 = (0)^2 = 0$.

Whereas the Right Hand Side (RHS), $a^2 + b^2 = (-1)^2 + (1)^2 = 1 + 1 = 2$.

$0 \neq 2$. *This is not an identity.*

b. $3 \cdot (a \cdot b) = 3a \cdot 3b$

Let $a = 1$ *and* $b = 1$.

LHS: $3 \cdot (a \cdot b) = 3 \cdot (1 \cdot 1) = 3 \cdot (1) = 3$

RHS: $3a \cdot 3b = 3(1) \cdot 3(1) = 3 \cdot 3 = 9$

$3 \neq 9$. This is not an identity.

4. Explain which rules were used at each step in solving this equation:

These properties are from the "Basic Rules of Arithmetic" Chap. 2 p. 30.

$3x + 7 = 22$	*Given*
$(3x + 7) + -7 = 22 + -7$	*add -7 to both sides*
$3x + (7 + -7) = 15$	**LHS:** *Associative Prop. Of Add.* **RHS:** *Subtraction fact*
$3x + 0 = 15$	*Additive Inverses exist*
$3x = 15$	*There is an additive identity*
$3^{-1} \cdot (3x) = 3^{-1} \cdot 15$	*Multiply both sides by* 3^{-1}
$(3^{-1} \cdot 3)x = 5$	**LHS:** *Associative Prop. Of Mult.* **RHS:** Multiplication fact
$1 \cdot x = 5$	*Multiplicative Inverse exists*
$x = 5$	*There is a multiplicative identity*

5. One student uses the following method to find the inverse of 15 mod 26. She lists all of the numbers that are 1 more than a multiple of 26 and finds one of those that is also a multiple of 15. Explain what the inverse is and why this method works:

1 — *15(7) = 105 = 26(4) + 1 ≡ 0 + 1 (mod 26). Therefore 15 and 7 are inverses.*

27 — ***Since 15*7*n≡n (mod 26)***

53

79

105 ←

131

Use this method to show how to find the inverse of 11.

Continue the numbers above to find a number that is a multiple of 11 AND that is equivalent to 1. These numbers are 157, 183, 209. 209 is a multiple of 11 so:
11(19) = 209 = 26(8) + 1 = 1 mod 26. Therefore, 11 and 19 are inverses.

Another student finds the inverse of 15 mod 26 by first listing all of the multiples of 15. From that list he looks for numbers that are 1 more than a multiple of 26.

Use this method to show how to find the inverse of 19 mod 26

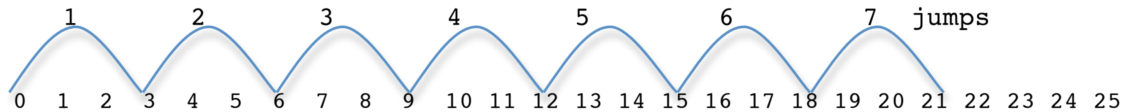
15	*19 = 19(1)*	*152 = 19(8)*
30	*38 = 19(2)*	*171 = 19(9)*
45	*57 = 19(3)*	*190 = 19(10)*
60	*76 = 19(4)*	*209 = 19(11)*
75	*95 = 19(5)*	
90	*114 = 19(6)*	
105 ←	*133 = 19(7)*	

209 = 26(8) + 1 = 1 mod26

19(11) ≡ 1 (mod 26) so they are inverses.

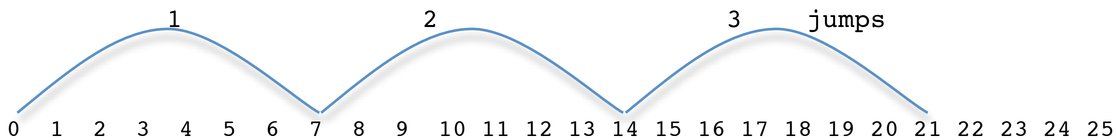
6. Use a number line and crickets to model the division statement. Be sure to say what are the groups and objects and whether or you are using a "How many groups?" or a "How many objects in each group?" model.

$21 \div 3 = 7$



These shows 21 spaces in 7 groups(jumps) of 3 spaces in each jump, so **$21 \div 3 = 7$**

The next picture is not as good because it makes me start with the +7 cricket which is the answer. It is always better to model in such a way as you see what the answer is.



This shows how many spaces (objects) in each of 3 jumps (groups) making up a total of 21 spaces

7. How would you explain to a middle school student that a negative number times a negative number is positive number? Use the fact that (-5) x (-4) = 20 as an example. *We received three different good answers. Here is one of them:*

First, I will show that a positive number times a negative is a negative.

$5 \times 3 = 15$ *(arithmetic rule)*
$5 \times 2 = 10$
$5 \times 1 = 5$ — *on the RHS, each number decreases by 5,*
$5 \times 0 = 0$ — *so if we keep going, 5 - 5 = 0*
$5 \times (-1) = -5$ — *the next is -5: we know this from number line work*
$5 \times (-2) = -5 - 5 = -10$ — *I continue in this manner:*
$5 \times (-3) = -10 - 5 = -15$
$5 \times (-4) = -15 - 5 = -20.$ — *So a positive times a negative is negative.*
It would work the same for other numbers

Let's fix -4.
$4 \times (-4) = -20 + 4 = -16$ — *Because we now have <u>one less</u> group of -4.*
$3 \times (-4) = -16 + 4 = -12$
$2 \times (-4) = -12 + 4 = -8$ — *another way to see this is that*
$1 \times (-4) = -8 + 4 = -4$ — *we have already showed that*
$0 \times (-4) = -4 + 4 = 0$ — *positive times negative is negative*
$(-1) \times (-4) = 0 + 4 = 4$ — *but this time the pattern continues*
$(-2) \times (-4) = 4 + 4 = 8$ — *up the number line: -8 -4 0 4 8 12*
$(-3) \times (-4) = 8 + 4 = 12$
$(-4) \times (-5) = 20$

8. Explain how to encrypt using the affine cipher (7,13):

First encode your message to numbers. Then take each number of the message and multiply by 7 and add 13. Change back to letters if you wish.

Explain how to decrypt a message that was encrypted using the affine cipher key=(7,13)

If the message is in Ciphertext, then encode it to numbers. Then subtract 13 from each number and multiply by the inverse of 7, which, as determined in question 5, is 15.

Encrypt or decrypt the following words as directed:
Computations not shown

Encrypt using AFFINE CIPHER, key = (7,13)

v	e	r	y	
21	04	17	24	*encode*
				Times 7
01	15	2	25	+ 13
E	P	C	Z	

This word was encrypted using the Affine Cipher, key = (7,13)
Decrypt:

n	i	c	e
A	R	B	P

9. Encrypt the word "blue" with two different multiplicative ciphers as directed:
Computations not shown

Encrypt using
Multiplicative cipher, key = 25

b	l	u	e
25	15	6	22

Encrypt using
Multiplicative cipher, key = -1

b	l	u	e
25	15	6	22

What do you notice? Explain.
They are the same because 25 is equivalent to -1 (mod 26). They are on the same spoke of the mod 26 wheel.

EXTRA CREDIT, but only if you have time. **Name:**

DO NOT USE CALCULATORS. Using the fact that 13 x 17 = 221, complete the following number facts in your head. In each case explain how you got the answer

Explanations varied, depending on what you are thinking that your audience already knows. Most people replied with procedural answers about moving decimal points. Of course, as future elementary teachers we want explanations the explain concepts. The following explanations use that we know what happens to decimal points when multiplying or dividing by 10. (Do you know how to explain <u>those rules</u> in terms of the meaning of multiplication and division??)

13 x 170 = $13 \times (17 \times 10) = (13 \times 17) \times 10 = 221 \times 10 = 2210$

note the use of the associative rule of multiplication

1.3 x 17 = $(13 \times 0.1) \times 17 = 13 \times (0.1 \times 17) =$
$13 \times (17 \times 0.1) = (13 \times 17) \times 0.1$

note the use of the associate and commutative rules of multiplication

0.13 x 1.7 = $(13 \times 0.01) \times (17 \times .1) = (13 \times 17) \times (0.01 \times 0.1) = 221 \times 0.001$

note the use of the associate and commutative rules of multiplication

2210 ÷ 13 = $\dfrac{2210}{13} = \dfrac{221 \cdot 10}{13} = \dfrac{13 \cdot 17 \cdot 10}{13} = 17 \cdot 10 = 170$

note the use of equivalent fractions (cancelling common factors in numerator and denominator)

22100 ÷ 1.7 = $\dfrac{22100}{17 \cdot \frac{1}{10}} = \dfrac{221 \cdot 100 \cdot 10}{17} = \dfrac{13 \cdot 17 \cdot 1000}{17} = 13 \cdot 1000 = 13000$

How do you explain how the $\frac{1}{10}$ *in the denominator becomes a* 10 *in the numerator?*