Circumference vs. Diameter

Teacher Lab Discussion

Overview

Length of a Curved Line

The perimeter of a circle, the distance around its "outside," is called the circumference. All the length measurements so far have involved the straight line distance between two points. How then would you measure the circumference of the circle shown in Figure 1? One way is to carefully lay a string along the line, mark the ends, straighten the string, and then use a ruler to measure the distance between the marked points. Unfortunately, this process is tedious since it is hard to juggle a string into place along a curved line and, besides, one doesn't always have a string handy. Another way is to divide the line into a series of segments each of which appears to be straight. This is illustrated in Figure 2. Then you use a ruler to measure the length of each "straight" line segment and add up the results. Next to a few of the segments we have written our estimate of their length. Why don't you try a few? Clearly, this is also tedious, especially if the circle is large, and there is always an error in replacing a true arc by an equivalent straight line no matter how small the segment. Still, we do want to find the relationship between the circumference (C) and the circle's diameter (D), so we need a simple, child-oriented method for finding the circumference that is quick and accurate.

Figure 1

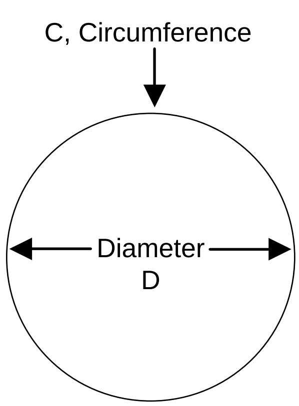
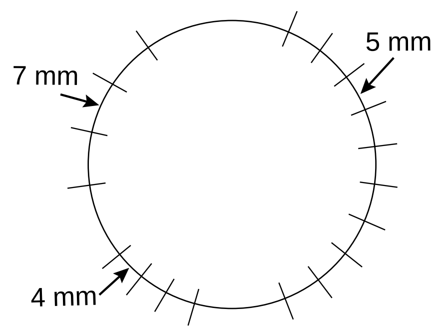


Figure 2



A nice way of finding C is shown in Figure 3. The child rolls a cylindrical object like a soft drink or orange juice can so that the object makes one full

Figure 3

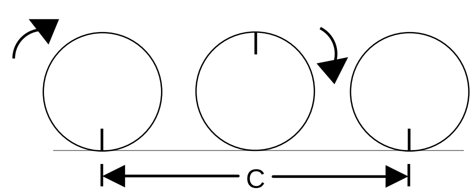


Figure 4

revolution. The distance rolled is precisely the circumference of the can. Voila! All you have to do is measure the distance rolled and you have the circumference.

As carried out in the elementary school, a few tips are in order. Try to gather rollers of three different sizes, small, medium, and large, so that you get a nice spread in diameters, which is our manipulated variable. Small should be about 3 to 4 cm (empty spools of sewing thread are perfect); medium about 6 to 8 cm (soft drink and orange juice cans are great); large, about 12 to 14 cm (a masking tape roll or coffee can will do) in diameter. Mark one end as shown in Figure 3. Have the children lay out and tape a piece of paper on the desk and draw a starting line on the paper. Then have them pick one small, one medium, and one large roller to use in the experiment. Have them mark the point where the cylinder has gone full circle. Since the start and stop lines are on the paper, they can easily measure the circumference, and you can check to see if there are any errors.

Picture, Data Table, and Graph

With the above in mind, our picture of the experiment is shown in Figure 4. The variables D and C should be clearly labeled. Either could be the manipulated variable but one usually chooses a can based on its diameter since it is the simpler linear measurement; so D will be treated as the manipulated and C the responding variable. Ask the children what variable they think should be the manipulated. See if they come up with similar reasoning.

In our drawing we have shown three stop marks. In **Question 1** we ask why. Being expert experimentalists by now the children should explain that "we need to make sure," "we always check by taking an average," "the can might skip," or "I might not start at the same spot each time." Taken together, these answers explain why we make three measurements and then average.

The data table for three well-spaced values of D is shown in Figure 5. As you go around the room ask the kids if their data "is proportional." Because of the value of D this is not so obvious, so they will have to think about it. Look at our first two data points. D goes from 3.9 cm to 7 cm— almost, but not quite, a factor of 2. C goes from 12.5 cm to 21.2 cm—again almost a factor of 2. So it does look as though the data is proportional.

How to tell for sure? Graph it, of course. Then if the graph is a straight line through (0,0), the variables are proportional.

Figure 5

Table I

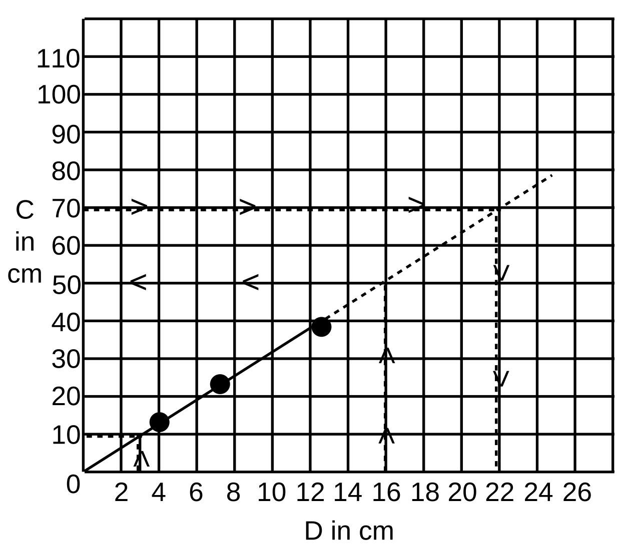
D in cm	C in cm			
	Trial 1	Trial 2	Trial 3	Average
3.9	12.5	12.3	12.6	12.5
7	21.2	21.6	21.6	21.6
12.3	39.5	38	41	39

Our data is graphed in Figure 6. And, indeed, it is a straight line through (0, 0) (**Question 2**). Actually (0, 0) is a good data point even without our doing the experiment. It is one of those gift points which we can derive logically. So in **Question 3** the children should answer, "yes," and "because as D goes to 0, the circle disappears and so C goes to zero too." And not only is the curve a straight line but what is even more interesting is that all the curves will have the same slope. If one child does his experiment with pop cans, another does his with jars, and someone else does his with coins, there will not be a family of curves, as with different types of bouncing balls, but a single straight line. The color of the objects, the material of which they are made, are all unimportant. A circle is a circle, or, more scientifically, the circumference vs. diameter curve is a universal curve, the same everywhere. Using the straight line, the children can interpolate or extrapolate to find the circumference or diameter of any size circle, even of the sun.

Comprehension Questions

The first three questions use the graph directly to find the answer. In **Question 4** if the diameter is 3 cm, then by interpolation we find C = 9.5 cm in **Question 5**. If the diameter is 16 cm, then by extending the curve, as shown in Figure 6, we find

Figure 6



that C = 50 cm. One can go in the other direction, and find D given C. In **Question 6** if the circle has a circumference of 70 cm, then its diameter, again by extrapolation, is close to 22 cm. The data technique is useful to biologists. We need to measure the diameters of trees without cutting them down. By circling the tree with a rope and measuring C, we can find D.

Since the curve is a straight line, then C/D should be the same for *all* pairs of data points. In **Question 7**, we ask the children to evaluate C/D for three values of C. They should get the following answers:

$$D = 6 \text{ cm}, \quad \frac{C}{D} = \frac{19 \text{ cm}}{6 \text{ cm}} = \frac{3.17 \text{ cm}}{1 \text{ cm}}$$

$$D = 12 \text{ cm}, \quad \frac{C}{D} = \frac{38 \text{ cm}}{12 \text{ cm}} = \frac{3.17 \text{ cm}}{1 \text{ cm}}$$

$$D = 18 \text{ cm}, \quad \frac{C}{D} = \frac{56 \text{ cm}}{18 \text{ cm}} = \frac{3.11 \text{ cm}}{1 \text{ cm}}$$

The ratios are all constant when compared to a common denominator of 1cm. These numbers should ring a bell. The ratio is close to 3.14159 which is the value of π. Thus

$$\frac{C}{D} = \pi$$

In fancy mathematics jargon, C/D is the slope of this curve. Thus, all C vs. D curves have the same *slope*, with a value of p. Even a Martian doing this experiment using units of "glugs" would still find C/D = $\pi \sim 3.14$. Indeed, p was a universal message sent into our galaxy several years ago when we made our first attempt to contact extraterrestrials.

The next several questions use proportional reasoning which is formal operational, compared to the previous questions which use the curve—a concrete operational action. We set up the problems with symbols, the ratio conveniently chosen from the curve or using p, then solve for the unknown quantity.

In **Question 8** we want to find the diameter of a circle if C = 120 cm. Then, properly set up we have

$$\frac{C}{D} = \frac{3.14\text{ cm}}{1\text{cm}} = \frac{120\text{ cm}}{D}$$

symbols from data

$$D = 120\text{ cm} \times \frac{1\text{ cm}}{3.14\text{ cm}} = 38.21\text{ cm}$$

If you do not want the children to use decimals, then round 3.14 off to 3! Although you lose accuracy (~5%), you call on the children's ability to multiply and divide in their heads or to use the calculators without decimals. But this is a wonderful chance for the children to use their calculators and to handle decimals.

In **Question 9** we look at a typical car tire with an inner diameter of 43 cm and an outer diameter of 71 cm. Solving for the inner circumference

$$\frac{C}{D} = \frac{3.14\text{ cm}}{1\text{ cm}} = \frac{C}{43\text{ cm}}$$

$$\therefore\ C = 43\text{ cm} \times \frac{3.14\text{ cm}}{1\text{ cm}} = 135.02\text{ cm}$$

The outer circumference is

$$C = 71\text{ cm} \times \frac{3.14\text{ cm}}{1\text{ cm}} = 222.94\text{ cm}$$

How far will the tire roll in one turn? In one turn it rolls its outer circumference or 222.94 cm.

As a nice addition to the problem, have the children measure the inner and outer diameters of the tires on their family car and on a neighbor's car and see which one rolls the farthest on one turn.

Besides being a practical problem, **Question 10** presents an interesting challenge to the children. A large hole has a circumference of 20 m, not 20 cm. What is D? New dimensions should not confuse them. Scaling up to meters we have

$$\frac{C}{D} = \frac{3.14\text{ cm}}{1\text{ cm}} = \frac{20\text{ m}}{D}$$

$$\therefore D = 20\text{ m} \times \frac{1\text{ cm}}{3.14\text{ cm}}$$

$$= 6.37\text{ m}$$

The units all cancel properly!

There is a famous picture taken in the 1920's of a group of 20 adults hugging a giant sequoia tree in California. Let's use that information in **Question 11** to find the tree's diameter. This is a more open-ended question so let the children come up with a solution. First they will have to estimate the arm span of a typical adult. Since an adult is around 2 meters tall, and from arm span ≅ height, we can estimate that arm span ~ 2 m.

Therefore, the circumference of the tree is ~ 40 meters. Using proportional reasoning

$$\frac{C}{D} = \frac{3.14\text{ cm}}{1\text{ cm}} = \frac{40\text{ m}}{D}$$

$$\therefore\ D = \frac{40\text{ m}}{3.14\text{ cm}} \times 1\text{ cm}$$

$$\sim\ 13\text{ meters}$$

Most of the children ride bikes, so **Question 12** is a chance for them to do a little practical pedaling. A bicycle wheel has a diameter of 64 cm. If you go 3000 meters (~2mi) how many turns does the wheel make. It's a two-step problem. First, the children must find the circumference:

$$\frac{C}{D} = \frac{3.14 \text{ cm}}{1 \text{ cm}} = \frac{C}{64 \text{ cm}}$$

$$\therefore C = 200.96 \text{ cm}$$

Next they determine how many turns it will make in 3000 m. First we change C into m so that

$$C \sim 2.01 \text{ m}$$

where we properly round off! Then one sets up the ratio of number of turns to distance traveled:

$$\frac{N_{\text{turns}}}{D_{\text{traveled}}} = \frac{1 \text{ turn}}{2.01 \text{ m}} = \frac{N_{\text{turns}}}{3000 \text{ m}}$$

$$N_{\text{turns to store}} = \frac{3000 \text{ m}}{2.01 \text{ m}} = 1492 \text{ turns}$$

Does it make any difference what the circle is made of? Since all kinds of objects were used in the experiment, the answer is no. Therefore, in **Question 13**, the answer is both cylinders would have the same circumference since they have the same diameter.

We conclude with a series of short problems from a wide variety of "practical" situations. You might want to treat some of these as homework or save one for a later exam question.

If they are going to make a profit, pizza manufacturers better know how to answer **Question 14**. For a 12-inch pizza,

$$\frac{C}{D} = \frac{3.14 \text{ inches}}{1 \text{ inch}} = \frac{C}{12 \text{ inches}}$$

$$\therefore C = \frac{12 \text{ inches} \times 3.14 \text{ in}\cancel{\text{ch}}\text{es}}{1 \text{ in}\cancel{\text{ch}}}$$

$$= 37.68 \text{ inches}$$

where we have used inches all the way, which is the American way!

Question 15 is for future restaurant owners and is a two-step logic problem. First we find the circumference of the table (in feet):

$$\frac{C}{D} = \frac{3.14}{1 \text{ ft}} = \frac{C}{8 \text{ ft}}$$

$$\therefore C = \frac{8 \text{ ft} \times 3.14 \text{ ft}}{1 \text{ ft}}$$

$$= 25.12 \text{ ft}$$

Now we find the number of people. Allowing 3 feet per person

$$\frac{N_{\text{people}}}{C} = \frac{1 \text{ person}}{3 \text{ feet}} = \frac{N}{25.12 \text{ feet}}$$

$$\therefore N = 25.12 \cancel{\text{ feet}} \times \frac{1 \text{ person}}{3 \cancel{\text{ feet}}}$$

$$= 8.37 \text{ people}$$

Notice units of feet cancel. We can't have a fraction of a person, so the table will hold 8 people.

In most countries, a $\frac{1}{2}$-mile race is ~800 m. If we want to build a track that covers that distance once around, then in **Question 16**

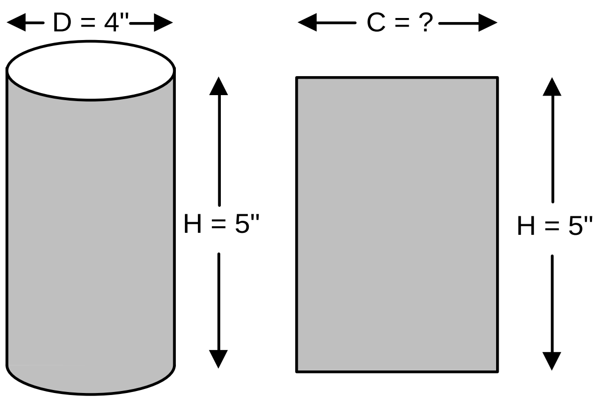
$$\frac{C}{D} = \frac{3.14 \text{ m}}{1 \text{ m}} = \frac{800 \text{ m}}{D}$$

Solving for D,

$$D = 800 \text{ m} \times \frac{1 \text{ m}}{3.14 \text{ m}}$$

$$= 255 \text{ m}$$

Figure 7



which is about $2\frac{1}{2}$ football fields (~100m) across. Maybe that's why most tracks are 400-meter ovals!

Sewing is a great art. In **Question 17** we present a problem which can be considered cost effective. Buy too little—disaster. Buy too much, expensive. So Anne figures if D = 6 inches, then

$$\frac{C}{D} = \frac{3.14 \text{ inches}}{1 \text{ inch}} = \frac{C}{6 \text{ inches}}$$

$$C = 18.84 \text{ inches}$$

Did you ever think about labels on cans? Who figures out how big they have to be? Well, **Question 18** is your chance to do that. Based on what the children have done, they should be able to "picture" the label before it goes on as a rectangle. Its height is 5 inches and its length is C, as shown in Figure 7 so,

$$\frac{C}{D} = \frac{3.14 \text{ inches}}{1\text{inch}} = \frac{C}{4 \text{ inches}}$$

$$C = 12.56 \text{ inches}$$

In **Question 19** we have a trundle wheel that clicks, i.e., comes around once every meter. The children should interpret this as, given C = 1 m, what is D?

$$\frac{C}{D} = \frac{3.14 \text{ m}}{1\text{m}} = \frac{1 \text{ m}}{D}$$

$$\therefore D = 0.318 \text{ m}$$
$$= 31.8 \text{ cm}$$

We conclude in **Question 20** with a project for the children. It should be easy to find a large circular garbage can lid. The one in back of our house is 61 cm across. Therefore, its circumference is

$$\frac{C}{D} = \frac{3.14 \text{ cm}}{1 \text{ cm}} = \frac{C}{61 \text{ cm}}$$

$$C = \frac{61 \text{ cm} \times 3.14 \text{ cm}}{1 \text{ cm}} = 191.5 \text{ cm}$$

The children now have to measure it either by the string technique or as in the lab, by rolling it. Say they find C = 196 cm. Was their prediction within 10% of the measured circumference? Well, 10% of 196 cm is

$$\Box = 196 \text{ cm} \times 0.1$$
$$= 19.6 \text{ cm}$$

which is well within

$$\Box = 196 \text{ cm} - 191.5 \text{ cm}$$
$$= 5.5 \text{ cm}$$

Summary

Circumference vs. Diameter is an all-important experiment. It deals with a universal curve which occurs over and over again in our daily lives. But beyond this marvelous relationship lies an important number, π, and the ability to use formal proportional reasoning to solve problems. And lots of problems there are as we explore C vs. D in all its ramifications.

If the children can do these problems, then they not only understand circumference vs. diameter, but they are also on their way to a higher level of formal thinking including ratios and algebra. Go for it!

Materials per Team

- 1 meterstick
- 3 cylinders: small, medium, large