Scale Drawings,

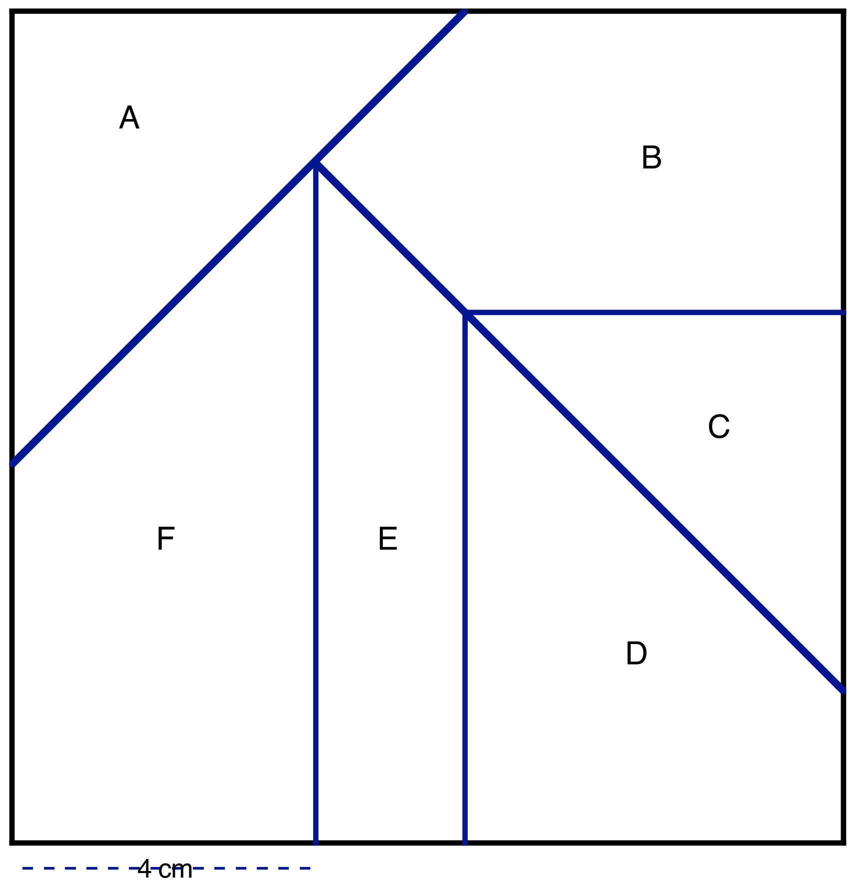
Proportional Reasoning

And

Similar Figures

Group Project: A Puzzle from Didactique

1. Assignment: Make an enlargement of this puzzle such that the length, that now measures 4 cm, will be 7 cm long.



Each person in your group should be assigned at least one piece to enlarge. Do not work on this together, but bring your piece, or pieces, back to class tomorrow. Use any tools. When your group has all the pieces together, assemble the larger puzzle.

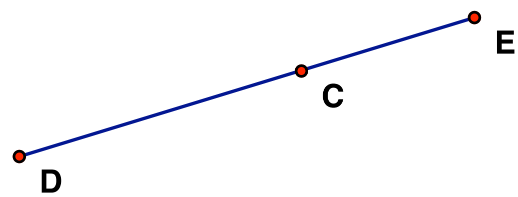
2. Write down your method here:

3. If the puzzle does not fit together, explain what went wrong.

Activity: Centimeters vs. inches

1. Measure the line segment $\overline{AB}$ using inches. Write your answer to the nearest $\frac{1}{16}$ of an inch and to the nearest tenth of an inch.

2. Measure the line segments $\overline{DE}$ and $\overline{DC}$ in centimeters then compute the length of $\overline{CE}$. Measure to the nearest millimeter. Confirm your answer by measuring the length of $\overline{CE}$.



3. Complete the second column of the table by using your ruler to draw line segments using inches and then measuring the segments in centimeters.

Find out the exact measurement in centimeters of 1 inch, and then fill in the third column

inches	centimeters measured	centimeters exact
1 in.		
1.5 in.		
2 in.		
2.5 in.		
3 in.		

4. What patterns do you see in the table?

5. Show how to use the meaning of multiplication to find the length of a 5-inch line segment in centimeters.

6. My height is 5′4″ what is my height in centimeters?

7. A meter stick is 100 centimeters long. Use division to find its measure in inches?

Use proportions to find the measurements in the next two problems:

9. Compute the measure in centimeters of the line segments in 1. Confirm by measuring.

10. Compute the measure in inches of the line segments in 2. Confirm by measuring.

Lesson 1 – Size Transformations

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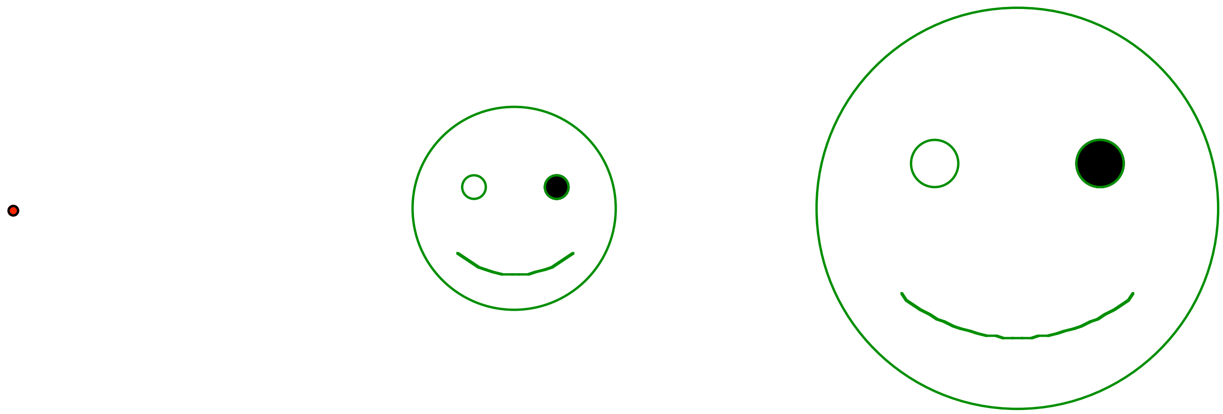


A scaling transformation changes the size of an object but preserves the shape of the object.. Notice that a line drawn through the point on the left through the small face ends up at the corresponding point on the large face. In fact for any such line the distance from the point to the large face is twice the distance from the point to the small face. Try this for several lines.

Also notice that the distance between any two points in the large face – say the distance between the center of the eyes – is twice that of the corresponding distance on the small face.

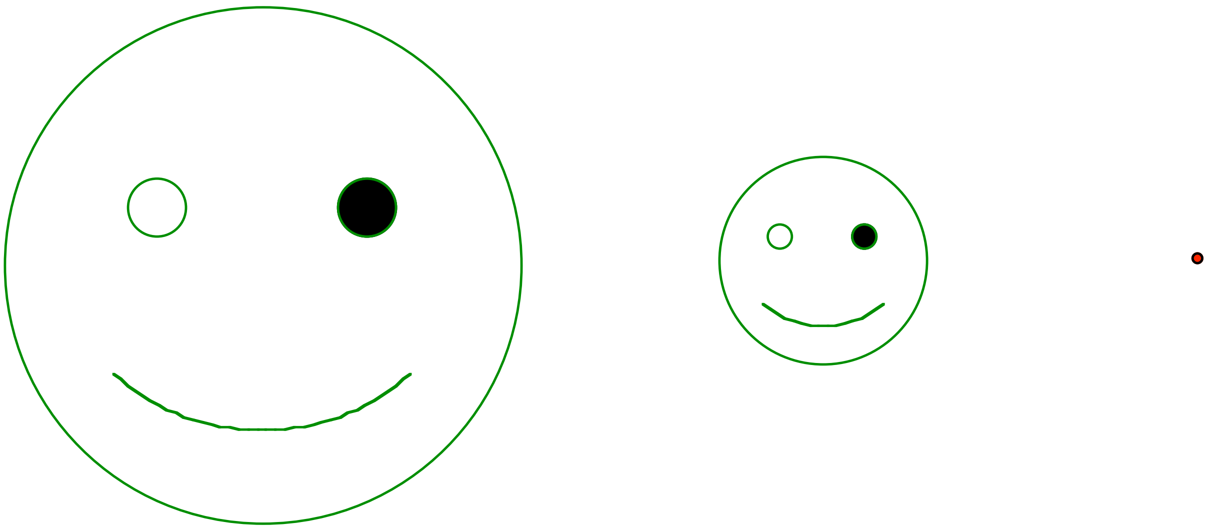
The small face has been scaled to twice its size.

One way to construct a figure that is twice the size is to use rubberbands. Link two rubberbands of equal size. Fasten one end of the double rubberband to the point on the left. Fix a pencil to the other end. Move the pencil to draw the enlarged smiley while tracing the original figure with the knot that links the two rubberbands.

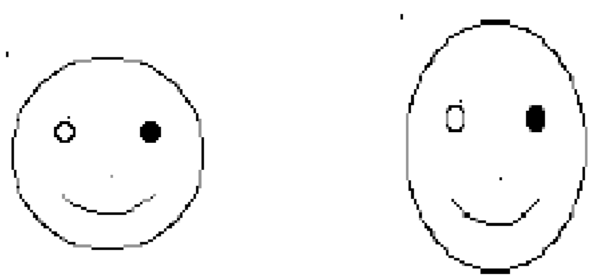
Iif you are left handed, turn the page upside down

A size (or scaling) transformation
enlarges (or shrinks) a figure by a factor of k and preserves all angle measurements.

Example 1: The large face on the left has been shrunk by a factor of 2.5 (so $k = \frac{1}{2.5} = 0.4$).



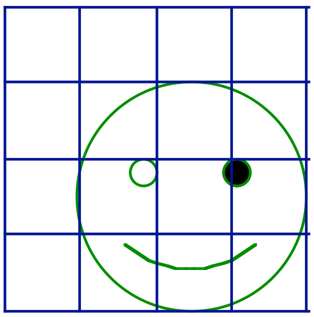
Example 2: This is not a size transformation:

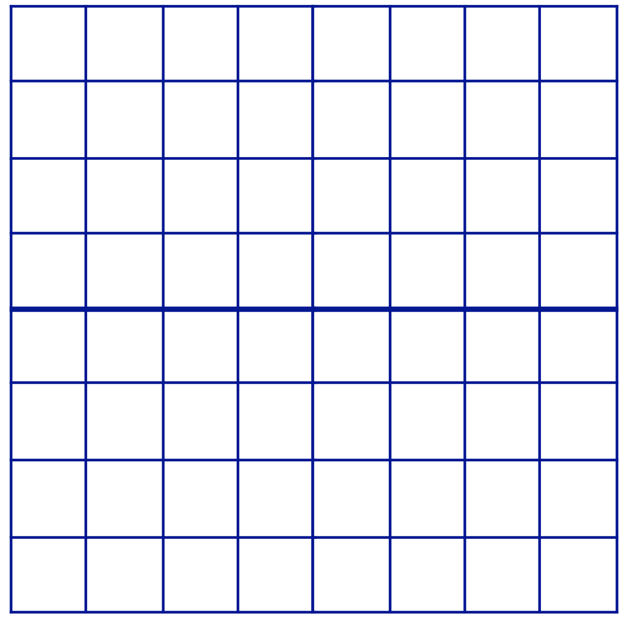


The face has been stretched vertically but not horizontally. It did not retain its shape. The circles are no longer circles and the smile is exaggerated.

Another way to think about size transformations is to think about the object on a grid.

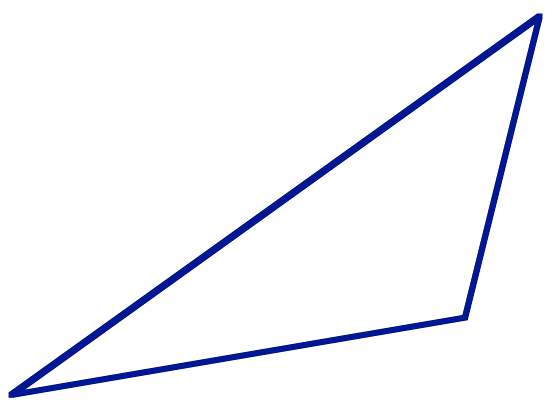
`Enlarge the smiley face by drawing it into a grid that is twice as large`



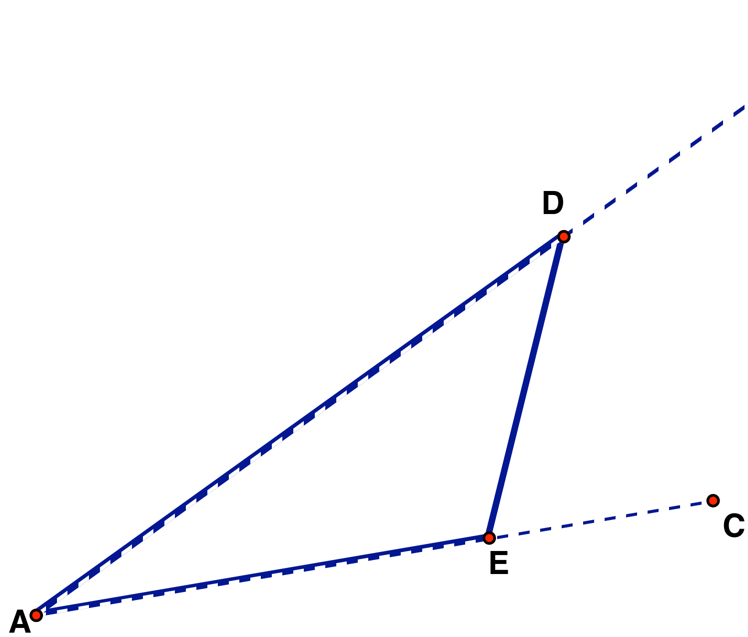


Activity: Practice with Size Transformations

Use both methods to make various sizes of this triangle.



Here is another way to enlarge a triangle. In this example we enlarge the triangle by a factor of 1.5. Pick one of the vertices and extend the two sides until the required length.



Verify by measuring that **AB** is 1.5 times **AD** and that AC is 1.5 times **AE**

Connect **B** and **C** and verify that **BC** is 1.5 times **DE**.

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Try making several more enlargements of the triangle using this method.
Compare with your other examples.
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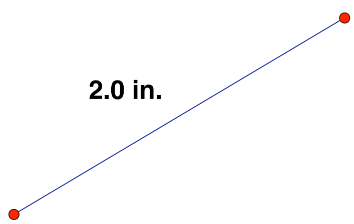
All of these triangles are said to be similar to each other: sides are in proportion and all the angles are the same.

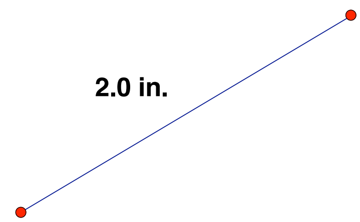
Making scale drawings

1. Anya has a garden and she wants to make a scale drawing of it. The garden is in the shape of a triangle. She measures each side. Her garden is 20 feet, 15 feet, 10 feet.

She draws a line segment 2 inches long to represent the side that is 20 feet long.
Finish the scale drawing for her.

Two triangles have the same shape when corresponding sides are in proportion.





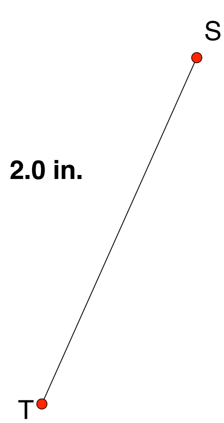
This drawing does not satisfy Anya it is too small.
Suppose she wants the 2.0 in. segment to represent the 10-foot side of the garden.
Finish this scale drawing for her.

What is the prorportion between the first and the second scale drawing?

2. Anya has another garden and she wants to make a scale drawing of it. The garden is in the shape of a triangle. She measures each angle. The three angles measure 40°, 60°, 80°.

She draws a line segment 2 inches long. The 40°-angle is at point S and the 60°-angle is at point T.
Finish the scale drawing for her.

Two triangles have the same shape when corresponding angles are equal.

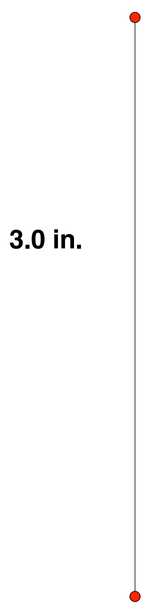


If the side of the garden represented by $\overline{ST}$ is actually 20 feet long, what are the lengths of the other sides of the garden?

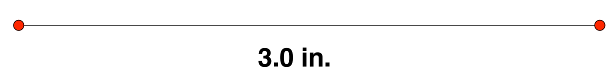
Rectangle problems

3. Anya has a garden and she wants to make a scale drawing of it. The garden is in the shape of a rectangle. She measures each side. Her garden is 20 feet x 30 feet.

She draws a line segment 3 inches long. Finish the drawing for her.

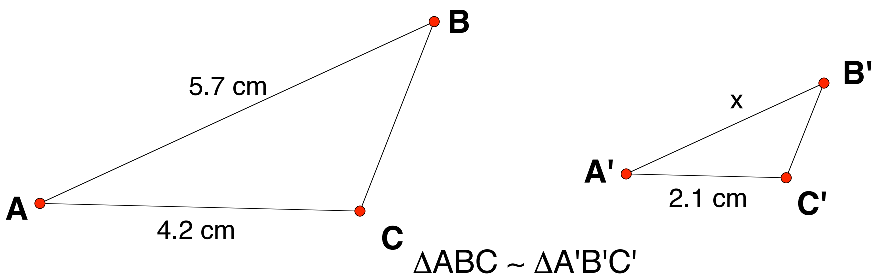


4. Suppose the 3.0 inch line is suppose to represent the short side of the garden. Finish the scale drawing for her.



Using similarity to solve problems

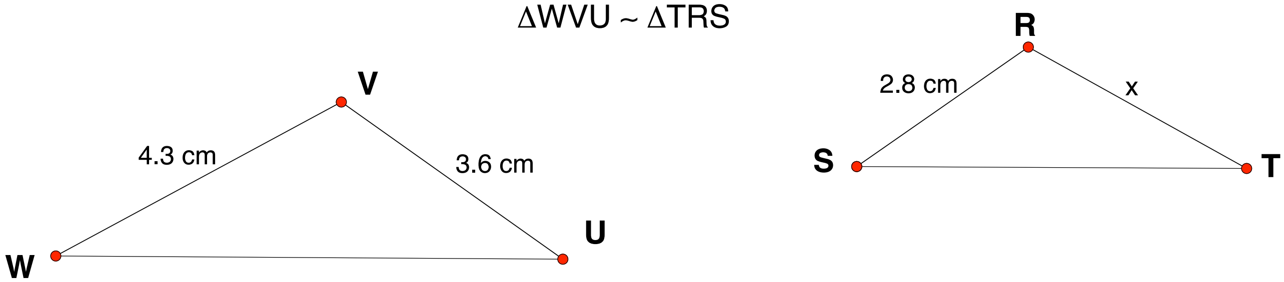
1. Find x. The drawing is exact to 0.1 cm so you may check your answer by measuring.



$\Delta ABC \sim \Delta A'B'C'$

2. Find x. The drawing is exact to 0.1 cm so you may check your answer by measuring.

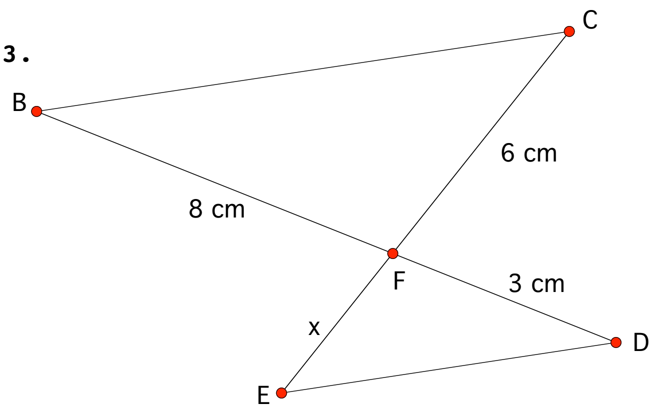
$\Delta WVU \sim \Delta TRS$



Using similarity to solve problems

In the next two problems you must find the missing length, x. Find x by first identifying similar triangles. Then write and solve proportions.

3.



$\overline{BC} \parallel \overline{ED}$

$m(\overline{CF}) = 6$ cm
$m(\overline{BF}) = 8$ cm
$m(\overline{DF}) = 3$ cm

What is the $m(\overline{EF})$?

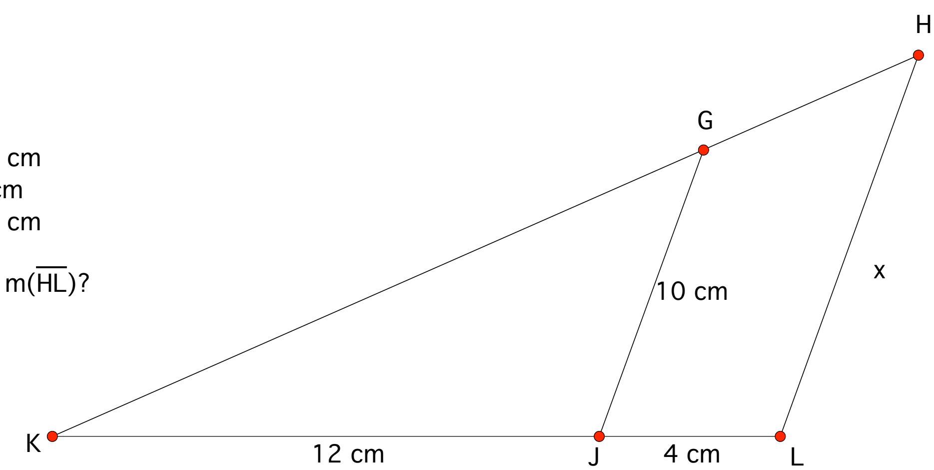
Complete this statement:

ΔBFC ~ Δ_______ because . . .

$\overline{GJ} \parallel \overline{HL}$

4. $m(\overline{GJ}) = 10$ cm
$m(\overline{JL}) = 4$ cm
$m(\overline{KJ}) = 12$ cm

What is the $m(\overline{HL})$?



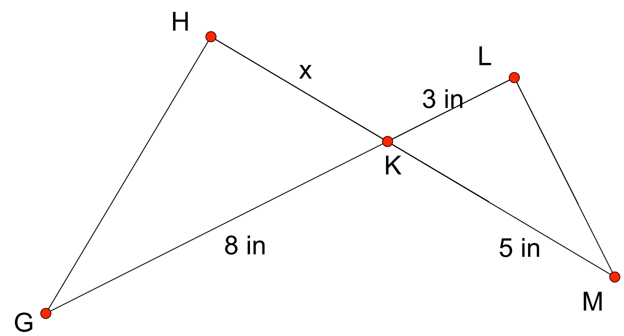
Complete this statement:

ΔKJG ~ Δ_______ because . . .

5.

m∠GHK = 90°

m∠MLK = 90°

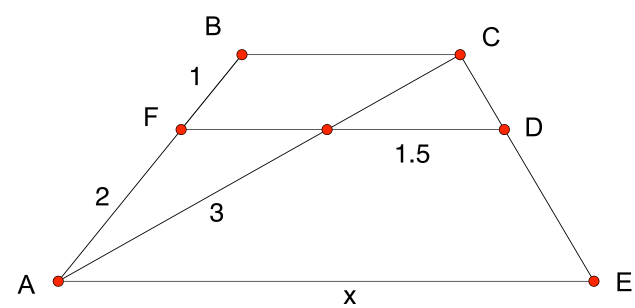


6. ABCE is a trapezoid.

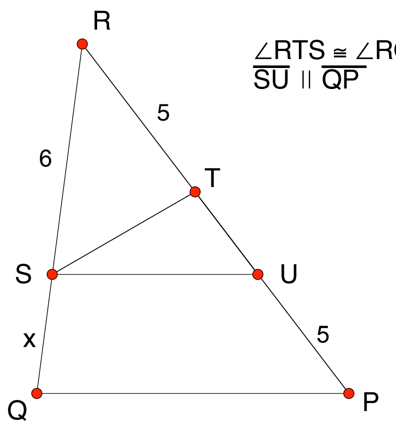
$\overline{FD} \parallel \overline{BC}$

$\overline{AC}$ is a diagonal of ABCE

Find x.



7. Identify three triangles that are similar to each other and write proportionality statements to find x.

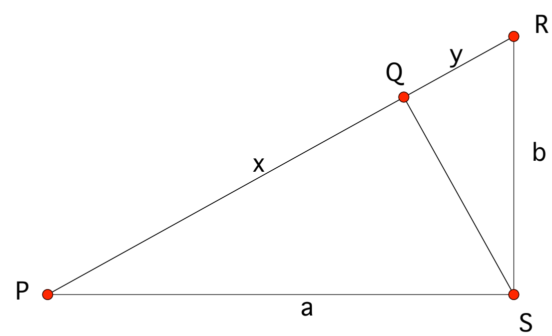


$\angle RTS \cong \angle RQP$
$\overline{SU} \parallel \overline{QP}$

8. Find three similar right triangles in this picture. Write down 2 different proportional statements involving the variables, a, b, c, x, y.

$\overline{QS} \perp \overline{PR}$
$\overline{PS} \perp \overline{RS}$

$m(\overline{PR}) = c$ (not shown in picture. It is the same as x+y)
$m(\overline{PQ}) = x$
$m(\overline{QR}) = y$
$m(\overline{PS}) = a$
$m(\overline{RS}) = b$



Examples and Counter-examples about similar figures

9. Answer the following questions **TRUE** or **FALSE**. If true give a reason why; if false give a counterexample.

TRUE or FALSE Any two equilateral traingles are similar.

TRUE or FALSE Any two equilateral traingles are congruent.

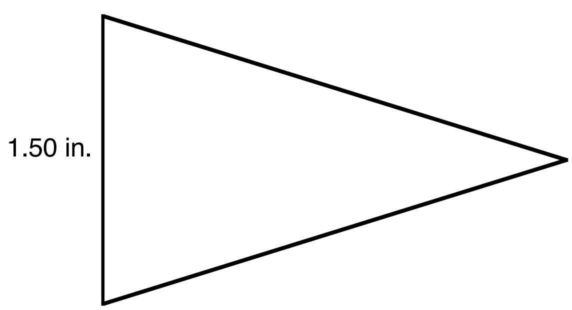
TRUE or FALSE Any two right traingles are similar.

TRUE or FALSE Any two rectangles are similar.

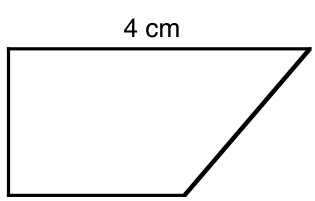
TRUE or FALSE Any two regular hexagons are similar.

TRUE or FALSE Any two hexagons are similar.

10. Construct a triangle that is similar to this isoceles triangle but the base is 1 inch long.



11. Construct a trapezoid that is similar to this trapezoid but the length that is 4 cm. is now 7 cm.

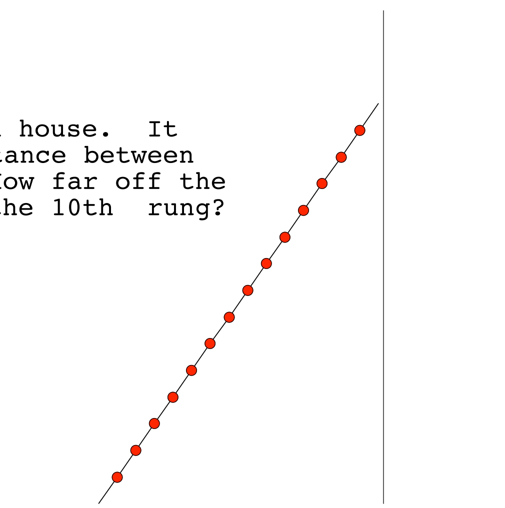


More problems with using similar triangles

Draw (or complete) a picture for each problem.

12. At 4:00 pm the shadow of a tree measures 5 feet 7 inches and the shadow of a 6'2" person measures 28 inches. How tall is the tree?

13. A 20 foot ladder is leaning against a house. It reaches a height of 17 feet. If the distance between each rung of the ladder is 16 inches. How far off the ground are you when you are standing on the 10th rung?



Dimensional Analysis

Cooks, carpenters, engineers , chemists, seamstresses and nurses, all need to be able to change from one unit of measurement to another in their work. In this section, we discuss a method of converting units called dimensional analysis and see examples of converting different units of length, area, volume, time, and speed.

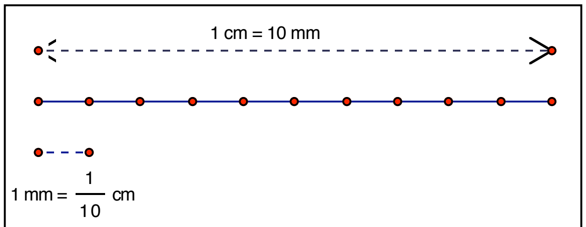
If you want to change the unit of measure, you can sometimes use the **meaning of multiplication**. For example, we know that 36 inches = 1 yard since 12 inches = 1 foot and 3 feet = 1 yard. *We have* 3 *groups (feet) with* 12 *objects (inches) in each group,*

So 3 feet X 12 inches in each foot = 36 inches

We often write $\frac{\text{inches}}{\text{foot}}$ instead of "inches per foot". With this notation, it looks like the feet and foot cancel each other as if they were like numbers in a fraction problem.

$$3\text{ feet X } 12\,\frac{\text{inches}}{\text{foot}} = 36\text{ inches}$$

However, you cannot always use the definition of multiplication. Sometimes it is necessary to use the **meaning of division**. For example, to express 240 mm as some number of cm, we figure out how many groups of 10 mm there are in 240 mm.



picture is enlarged from actual size

Here each group is 1 cm that consists of 10 mm. Writing this as a division problem looks like

$$240\text{ mm} \div 10\,\frac{\text{mm}}{\text{cm}}.$$

This can look like a multiplication problem, when we remember that division by 10 is the same as multiplying by $\frac{1}{10}$, so

$$240\text{ mm} \div 10\,\frac{\text{mm}}{\text{cm}} = 240\text{ mm} \times \frac{1\text{ cm}}{10\text{ mm}} = 24\text{ cm}.$$

Normally, this kind of problem would just be done in your head. Here is another example. How many feet in 30 inches? Think of this as a "how many groups?" division problem, where each group is one foot – a group of 12 inches. How many groups of 12 inches are there in 30 inches? The answer is 30 divided by 12.

$$30\text{ inches} \div 12\,\frac{\text{inches}}{\text{foot}} = 1\frac{1}{2}\text{ feet}$$

by writing this expression as multiplication by the reciprocal of 12, we again see how the units (this time the inches) appear to "cancel out"

$$30 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} = 1\frac{1}{2} \text{ feet}$$

One method for changing any unit measurement to another, which takes advantage of this apparent canceling of units, is called **dimensional analysis.** Dimensional analysis uses the idea of a *unit ratio*.

> **Definition:** A *unit ratio* is a fraction that has the value of 1 if both the numerator and denominator are expressed in the same units.

For example, $\frac{1 \text{ foot}}{12 \text{ inches}} = \frac{1 \text{foot}}{1 \text{foot}} = 1$.

You know that a fraction has many names. For example, $\frac{1}{2} = \frac{2}{4} = \frac{20}{40}$. One way to find different names for a fraction is to multiply one form by another fraction representing the value of 1.

$$\frac{1}{2} \cdot \frac{2}{2} = \frac{2}{4} \qquad \text{or} \qquad \frac{1}{2} \cdot \frac{10}{10} = \frac{20}{40}$$

Dimensional analysis tells us to multiply any measurement by a unit ratio, using the given unit of measure to guide our choice of unit ratio, to express the measurement in an equivalent form.

For example, suppose that you want to change

180 inches to ____ feet.

We know that 12 inches = 1 foot. Both $\frac{12 \text{ inches}}{\text{foot}}$ and $\frac{1 \text{ foot}}{12 \text{ inches}}$ are unit ratios, because numerator and denominator of each represent the same length. However, if we multiply: $180 \text{ inches} \times \frac{12 \text{ inches}}{\text{foot}}$, we get a more complicated, not meaningful expression. However, if, using the meaning of multiplication, we multiply by $\frac{1 \text{ foot}}{12 \text{ inches}}$, the unit “inches” cancel, and we have

$$180 \text{ inches X } \frac{1 \text{ foot}}{12 \text{ inches}} = 15 \text{ feet}$$

Since the commutative property applies to the operation of multiplication, dimensional analysis can be done in different orders and even in several steps. In each of these examples, notice how the units appear to “cancel.”

Example 1: 156.92 cm = ____ m

$$156.92 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} = 1.5692 \text{ m}$$

Example 2: 3467 inches = ______ yds

$$3762 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} = 313.5 \text{ feet and then}$$

$$313.5 \text{ feet} \times \frac{1 \text{ yard}}{3 \text{ feet}} = 104.5 \text{ yards}.$$

Another way to do this is in one long string

$$3762 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} \times \frac{1 \text{ yard}}{3 \text{ feet}} = 3462 \div 12 \div 3 \text{ yards} = 104.5 \text{ yards}$$

Example 3: 54 ft^2 = ____ yd^2

We do not need new facts when using dimensional analysis to convert area measurements. One ft^2 is the area of a square that is 1 foot by 1 foot. As a unit of area we short cut and think of ft^2 as ft(ft) just as we can think of x^2 as x(x). Here are three methods to convert ft^2 to yd^2.

$$\text{Method 1: } 54 \text{ ft}^2 \times \left(\frac{1 \text{ yd}}{3 \text{ ft}}\right) \times \left(\frac{1 \text{ yd}}{3 \text{ ft}}\right) = 6 \text{ yd}^2$$

$$\text{Method 2: } 54 \text{ ft}^2 \times \left(\frac{1 \text{ yd}}{3 \text{ ft}}\right)^2 = 6 \text{ yd}^2$$

A third method, uses the fact that 1 yd^2 is the same area as 9 ft^2

$$\text{Method 3: } 54 \text{ ft}^2 \times \frac{1 \text{ yd}^2}{9 \text{ ft}^2} = 6 \text{ yd}^2$$

1. Assignment: Draw a picture and make a convincing argument that shows that 1 yd^2 is 9 ft^2.

Example 4: 16 m = ______ ft

In order to solve example 4, we must know another conversion fact. It is not enough to know some relationships among the metric measurements and to know some relationships among the English measurements. We must also know at least one relationship between a metric measurement and an English measurement. A convenient conversion to use is 1 inch = 2.54 cm.

Method 1:

$$16 \text{ m} \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right) \times \left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right) \times \left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \approx 52.4934 \text{ ft}$$

(to 4 decimal place accuracy)

Method 2: Some people remember that 1 foot = 30.48 cm.

$$16\text{ m} \times \left(\frac{100\text{ cm}}{1\text{ m}}\right) \times \left(\frac{1\text{ ft}}{30.48\text{ cm}}\right) \approx 52.4934\text{ ft}$$

The following example shows how to use dimensional analysis to convert speeds, by using unit fraction s for both distance and time units.

Example 4: $16\ \frac{\text{feet}}{\text{sec}} = \_\_\_\_\_\_\ \frac{\text{meters}}{\text{min}}$

$$16\frac{\text{feet}}{\text{sec}} \times \frac{12\text{ inches}}{1\text{foot}} \times \frac{1\text{meter}}{30.48\text{ inches}} \times \frac{60\text{ sec}}{1\text{min}} = 377.953\frac{\text{meters}}{\text{min}}$$

2. Use dimensional analysis, to convert the following units. You may have to do some research to find appropriate unit ratios if you don't know ones that will do the problem.

17 yards = ______ cm

25 miles = ______ kilometers

12 ft^2 = ______ in^2

107 cm^3 = ______ ft^3

37 weeks = ______ hours

$60\ \frac{\text{miles}}{\text{hour}} = \_\_\_\_\_\_\ \frac{\text{feet}}{\text{sec ond}}$

24 pints = ______ gallons

17 tablespoons = _______ cups

1 year = _______ minutes

45 ft^2 = ______ yd^2

3. Make a list of convenient unit ratios that you might want to commit to memory.

The unit ratios we've used so far are given to us. If we didn't know how many centimeters in one inch, we could look it up in a book or on the internet. In the following problem the unit ratios are derived from the information given in the problem.

4. There are about 1 billion people in China. If they lined up four to a row and marched past you at the rate of 25 rows per minute, how long would it take the parade to pass you:

$$10^9 \text{ people} \times \frac{1 \text{row}}{4 \text{people}} \times \frac{1 \text{ minute}}{25 \text{rows}} = 10^7 \text{ minutes}$$

5. Explain why $\frac{1 \text{row}}{4 \text{people}}$ is a unit ratio.

Because it will take so long for all the Chinese people to march past, minutes is not the best unit to use. We can use dimensional analysis to convert minutes to years:

$$10^7 \text{ minutes} \times \frac{1 \text{hour}}{60 \text{minutes}} \times \frac{1 \text{ day}}{24 \text{hours}} \times \frac{1 \text{year}}{365 \text{days}} \approx 19 \text{years}$$

6. Check this calculation with your own calculator and give the answer rounded to 3 decimal points. Do the calculation without retyping any numbers into your calculator.

7. Challenge: Give the answer to this example in years, days and hours instead of using a decimal fraction of a year.

More problems

8. Which is faster – a car traveling at 60 mph or a cheetah running at 25 meters per second?

9. Will Pete have enough paint to cover one wall of his room if he buys a can of paint that covers 30 sq ft and his wall has an area of 8 sq yd?

10. If gasoline in "A-land" costs 72.9 A¢ per liter, is it more expensive than gasoline in Chicago? Note that 1 A¢ = .79 US¢ and 1 gal = 3.7854 L

11. Prior to conversion to a decimal monetary system, the United Kingdom used the following coins.

1 pound = 20 shillings
1 shilling = 12 pence (NOTE: pence is the plural of penny)
1 penny = 2 half-pennies = 4 farthings

a) How many pence were there in a pound?

b) How many half-pennies were in a pound?

c) How many farthings were equal to a shilling?

12. A restaurant chain has sold over 80 billion hamburgers. A hamburger is about one-half inch thick. If the moon is 240,000 miles away, what percent of the distance to the moon is the height of a stack of 80 billion hamburgers?

13. A teacher and her students established the following system of measurements for the Land of Names.

1 jack = 24 jills
1 james = 8 jacks
1 jennifer = 60 james
1 jessica = 12 jennifers

Complete the following table.

	Jill(s)	Jack(s)	James(s)	Jennifer(s)	Jessica(s)
Jill =	1				
Jack =		1			
James =			1		
Jennifer=				1	
Jessica=					1