Antiderivatives

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1. What does it mean for $F(x)$ to be an *antiderivative* of $f(x)$?
2. Let $f(x)$ be a function and $F(x)$ an antiderivative. Prove that $F(x)+C$ is also an antiderivative of f, where C is any constant. Why do we always refer to *an* antiderivative of a function, as opposed to *the* antiderivative?

In problems 3-5, if true prove it! If false, give a counterexample.

3. True or false: if $F(x)$ is an antiderivative of $f(x)$ and $G(x)$ is an antiderivative of $g(x)$, then $F(x)+G(x)$ is an antiderivative of $f(x)+g(x)$?
4. True or false: if $F(x)$ is an antiderivative of $f(x)$ and c is a constant, then $cF(x)$ is an antiderivative of $cf(x)$?
5. True or false: if $F(x)$ is an antiderivative of $f(x)$ and $G(x)$ is an antiderivative of $g(x)$, then $F(x)G(x)$ is an antiderivative of $f(x)g(x)$?

In problems 6-14, find an antiderivative for $f(x)$. A good strategy is to do the following:

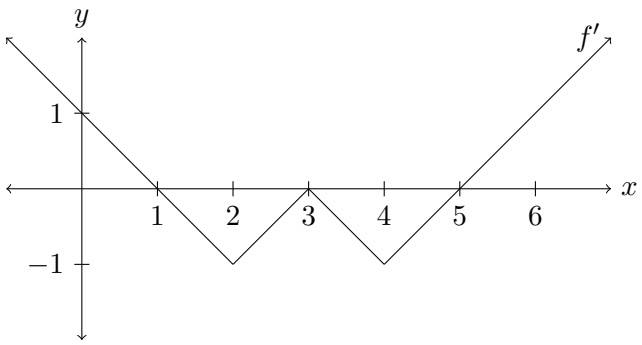
(a) Take a guess.

(b) Check your guess by taking its derivative. If you get $f(x)$, congratulations: your guess was correct and you've found an antiderivative of $f(x)$! Otherwise, revise your guess and repeat.

6. $f(x) = x^4$
7. $f(x) = \sin(2x)$
8. $f(x) = 3\sec^2 x$
9. $f(x) = \frac{3}{x^2} + 4x^2 - 1$
10. $f(x) = (5x+3)^4$
11. $f(x) = \frac{1}{2x} + e^{2x}$
12. $f(x) = \frac{\sin x - 1}{\cos^2 x}$
13. $f(x) = \frac{1}{\sqrt{1-x^2}}$
14. $f(x) = \frac{1}{16x^2+25}$

15. Let $f(x) = x^5 - 2x^{-2} + 1$. Find an antiderivative $F(x)$ such that $F(1) = 0$. Hint: use problem 2.
16. Find a function $f(x)$ such that $f'(x) = 6\sin(3x)$ and $f(\pi/6) = 6$.

17. Find the equation of a function f whose *derivative* is shown here:



Note: the derivative f' is defined everywhere, so your function should be differentiable (and therefore continuous) everywhere.