The chain rule

TA: Sam Cole

9/30/14

Use the chain rule to calculate $\frac{dy}{dx}$ for the following functions:

- 1. $y = (5x^2 + 11x)^{20}$ 2. $y = \sqrt{x^2 + 1}$ 3. $y = e^{\sqrt{x}}$ 4. $y = \tan(5x^2)$ 5. $y = \left(\frac{3x}{4x+2}\right)^5$ 6. $y = \cos^4 x + \sin^4 x$
- 7. $y = 2^x$. Hint: write $2^x = e^{\text{something}}$.
- 8. It's 7:05 p.m. on Wednesday, October 22, and you're in the middle of midterm 2. So far it's going OK, but then you get to a problem which requires you to use the quotient rule, and—oh no!—you can only remember the product rule and the chain rule! Use the product rule and the chain rule to derive the quotient rule. I.e., if h(x) = f(x)/g(x), derive the formula for h'(x). Hint: $h(x) = f(x) \cdot \frac{1}{g(x)}$. (Note the *correct* use of the word 'derive.')
- 9. Let h(x) = f(g(x)) and p(x) = g(f(x)). Use the table to compute the following:

(a) $h'(3)$				(c) $p'(4)$	(e) $h'(5)$
(b) $h'(2)$				(d) $p'(2)$	
$x \mid 1$	2	3	4	5	

f(x)	0	3	5	1	0
f'(x)	5	2	-5	-8	-10
g(x)	4	5	1	3	2
g'(x)	2	10	20	15	20

10. Let p(x) = f(g(x)) and q(x) = f(g(h(x))).

- (a) What is the formula for p'(x) (this is just the statement of the chain rule using Newton notation)?
- (b) Find a formula for q'(x). Hint: q(x) = p(h(x)); use the chain rule and part (a).

Use the chain rule multiple times to differentiate the following (your answer to problem 10b may help):

- 11. $\cos^4(7x^3)$ 13. $\tan(e^{\sqrt{3x}})$ 15. $\sqrt{x + \sqrt{x + \sqrt{x}}}$ 12. $\sin^5(\cos(3x))$ 14. $\sqrt{x + \sqrt{x}}$
- 16. Let f_n denote f composed with itself n times. This is called the *nth iterate* of f. So, for example, $f_1(x) = f(x)$, $f_2(x) = f(f(x))$, $f_3(x) = f(f(f(x)))$...you get the idea. Find formulas for the following:
 - (a) $f'_1(x)$ (c) $f'_3(x)$ (e) $f'_n(x)$
 - (b) $f'_2(x)$ (d) $f'_4(x)$