Derivatives

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1. Consider the function $f(x) = x^2$.

   (a) Sketch the graph of $f(x)$. On the same set of axes, draw the tangent line at $x = 1$ and the secant lines on the intervals $[1, 3]$, $[1, 2]$, and $[1, 1.5]$. What appears to be happening to the secant line as the intervals get smaller?

   (b) Give a formula for the slope of the secant line m_{avg} on the interval $[a, a + h]$ for *any* function $f(x)$. Hint: two points on the line are $(a, f(a))$ and $(a + h, f(a + h))$...

   m_{avg} is also the *average rate of change* of $f(x)$ on the interval $[a, a + h]$.

   (c) Fill in at least part of the following table for $f(x) = x^2$:

Interval	m_{avg}
$[1, 3]$	
$[1, 2]$	
$[1, 1.5]$	
$[1, 1.1]$	
$[1, 1.01]$	
$\vdots$	

   What number do the values of m_{avg} appear to be approaching?

   (d) The *derivative* of a function $f(x)$ at $x = a$, denoted $f'(a)$, is the slope of the tangent line at $x = a$, or the instantaneous rate of change at $x = a$. You can think of this as the slope of the secant line or the average rate of change on an interval of length 0.

   Give a formula for $f'(a)$ for any function $f(x)$. Hint: use limits and your answer for part (b)!

   (e) Use your formula to compute $f'(1)$ for $f(x) = x^2$. Is your answer consistent with your answer for part (c)?

   (f) Give yourself a pat on the back...you now know what a derivative is!

2. You can find the equation for a line if you know the *slope* and a *point on the line*. Use this information to find the equation for tangent line to $f(x) = x^2$ at $x = 1$. The slope is given by $f'(1)$; what is a point on the line? Hint: use the *point of tangency*, the point where the tangent line touches the graph of $f(x)$.

3. Evaluate $f'(a)$ at the given value of a:

   (a) $f(x) = 8x$; $a = -3$.
   (b) $f(x) = 4x^2+2x$; $a = -2$.
   (c) $f(x) = 2x^3$; $a = 10$.
   (d) $f(x) = \frac{1}{\sqrt{x}}$; $a = \frac{1}{4}$.
   (e) $f(x) = \frac{1}{x^2}$; $a = 1$.
   (f) $f(x) = \frac{1}{x+1}$; $a = 1$.
   (g) $f(x) = 2\sqrt{x}$; $a = 25$.