Rules for derivatives

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1. Warmup: what is the equation for the line with slope 5 through the point $(2, 9)$? Suggestion: this is REALLY easy if you use *point-slope* form.

2. Let $f(x) = 3x^4 + 7x$.

   (a) Find the derivative $f'(x)$.

   (b) What is $f'(1)$? What is the geometric significance of this number?

   (c) Find the equation for the line tangent to $f(x)$ at $x = 1$. Hint: think about how you did problem **??**; what is the slope? What is a point on the line?

3. Let $f(x) = x^n$.

   (a) What is $f'(x)$?

   (b) What is $f''(x)$?

   (c) What is $f'''(x)$?

   (d) $\vdots$

   (e) What is $f^{(n)}(x)$ (the nth derivative)?

   (f) What is $f^{(m)}(x)$ for any $m > n$?

   (g) Now suppose $f(x) = x^n + g(x)$, where $g(x)$ is a polynomial of degree $< n$. *Prove* that your answer to part (e) remains unchanged. Hint: use parts (e) and (f) and the sum/constant multiple rules.

4. Convert from Newton to Leibniz notation or vice versa:

   (a) $f'(x)$, where $y = f(x)$.

   (b) $f'(x)$, without using another variable.

   (c) $\frac{d}{dx}(-3x^2 + 2)$.

   (d) $\frac{d}{dx} f(x)\big|_{x=2}$.

   (e) y''.

   (f) $f'''(2)$.

   (g) $\frac{d^n}{dx^n} f(x)$.

5. What is the verb which means 'to take the derivative of'? Hint: it is NOT 'derive'! Using 'derive' in place of the correct word will not hurt your grade, but it will make me die a little each time.

For the remaining problems, do the following:

(a) Find the derivative $f'(x)$.

(b) Evaluate $f'(a)$ for the given a.

(c) Find the equation for the line tangent to $f(x)$ at $x = a$.

6. $f(x) = \sqrt{x}$; $a = \frac{1}{4}$.
7. $f(x) = e^2$; $a = 987234786582$.
8. $f(x) = e^x$; $a = \ln 3$.
9. $f(x) = 6\sqrt{x} - 4x^3 + 9$; $a = 9$.
10. $f(x) = (2x + 1)(3x^2 + 2)$; $a = 1$.
11. $f(x) = \frac{x^2-1}{x-1}$; $a = 0$.
12. $f(x) = \frac{x-2}{\sqrt{x}-\sqrt{2}}$; $a = 0.01$.