The fundamental theorem of calculus

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In problems 1-6:

(a) Find an antiderivative $F(x)$ of the integrand $f(x)$. In other words, evaluate $\int f(x)dx$.

(b) Use the FTC to evaluate the definite integral.

1. $\int_{-2}^{2}(x^2-4)dx$
2. $\int_{0}^{\ln 8} e^x dx$
3. $\int_{0}^{4} x(x-2)(x-4)dx$
4. $\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}}$
5. $\int_{-\pi/2}^{\pi/2}(\cos x - 1)dx$
6. $\int_{1}^{2} \frac{3}{x}dx$

In problems 7-9:

(a) Sketch the graph of $f(x)$ on the given interval.

(b) Shade the region bounded by $f(x)$ and the x-axis.

(c) Find the area of the region. Note: areas should always be nonnegative!

7. $f(x) = 1/x; [1, 2]$
8. $f(x) = \cos x; [\pi/2, \pi]$
9. $f(x) = x^3 - 1; [-1, 2]$

Simplify:

10. $\frac{d}{dx}\int_{3}^{x}(t^2+t+1)dt$
11. $\frac{d}{dx}\int_{2}^{x^3}\frac{dt}{t^2}$ (Hint: chain rule)
12. $\frac{d}{dx}\int_{x}^{1}\sqrt{t^4+1}dt$ (Hint: reverse limits of integration)
13. $\frac{d}{dx}\int_{-x}^{x}\frac{dt}{t^2+1}$ (Hint: split into two integrals)
14. $\frac{d}{dx}\int_{e^x}^{e^{2x}}\ln t^2 dt$

15. Let $A(x) = \int_{-2}^{x} f(t)dt$ and $F(x) = \int_{4}^{x} f(t)dt$. Use the picture to evaluate the following:

(a) $A(-2)$ (c) $A(4)$ (e) $A(8)$ (g) $A'(0)$ (approximately)

(b) $F(8)$ (d) $F(4)$ (f) $F'(8)$