Definite integrals

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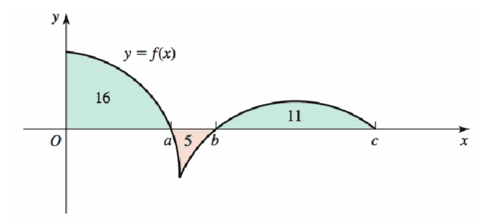
You man *not* use the fundamental theorem of calculus for this worksheet!

1. Suppose $f(x) \geq 0$ on $[a, b]$. What is the geometric meaning of $\int_a^b f(x)dx$? What if $f(x)$ is not ≥ 0 on all of $[a, b]$?

2. Sketch the graph of $f(x) = x^3$ on $[-1, 2]$. Then approximate the net area bounded by $f(x)$ and the x-axis using a left, right, at midpoint Riemann sum with 4 subintervals.

3. Use the picture to evaluate the integrals:



   (a) $\int_0^a f(x)dx$ (b) $\int_0^b f(x)dx$ (c) $\int_a^c f(x)dx$

4. Use geometry to evaluate $\int_0^4 \sqrt{16 - x^2}dx$.

5. Use properties of integrals to evaluate the following integrals, given that $\int_0^3 f(x)dx = 2$, $\int_3^6 f(x)dx = -5$, and $\int_3^6 g(x)dx = 1$.

   (a) $\int_0^3 5f(x)dx$ (b) $\int_3^0 f(x)dx$ (c) $\int_3^6 (3f(x) - g(x))dx$

6. Suppose $f(x) \geq 0$ on $[0, 2]$, $f(x) \leq 0$ on $[2, 5]$, $\int_0^2 f(x)dx = 6$, and $\int_0^5 f(x)dx = -2$. Evaluate the following:

   (a) $\int_2^5 f(x)dx$
   (b) $\int_0^5 |f(x)|dx$
   (c) $\int_2^5 4|f(x)|dx$
   (d) $\int_0^5 (f(x) + |f(x)|)dx$

7. Consider the function $f(x) = h(\frac{x}{b})^p$, where b, h, and p are constant.

   (a) Sketch the cases $p = 1, 2, 3$ on the interval $[0, b]$ on a single set of coordinate axes.

(b) What well known formula gives $\int_0^b f(x)dx$ when $p = 1$?

(c) Consider the case $h = b = 1$, $p = 2$, i.e. $f(x) = x^2$. Recall that

$$\int_0^1 x^2 dx = \lim_{n\to\infty}(\text{midpoint Riemann sum with } n \text{ subintervals}).$$

Use a midpoint Riemann sum with 2 subintervals to approximate $\int_0^1 x^2dx$. Now do the same using 3 subintervals, then 4 subintervals, then 5 subintervals, etc. What appears to be the value of $\int_0^1 x^2dx$?

(d) Make a conjecture about the value of $\int_0^b f(x)dx$ for h, b, and p in general. How does the area under $f(x)$ on $[0, b]$ compare with that of a $b \times h$ rectangle?

8. Consider $f(x) = x^2$ on $[0, 1]$.

(a) Write the formula for the right Riemann sum with n subintervals, using Σ notation.

(b) Evaluate $\int_0^1 x^2dx$ by taking the limit of the right Riemann sum as $n \to \infty$. Compare with your answer to 7c.

Hint: use the identity

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$