L'Hôpital's rule

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In problems 1-6:

(a) Write the indeterminate form you get when you try to evaluate the limit by substitution. It must be 0/0 or $\pm\infty/\infty$ in order to use L'Hôpital's rule.

(b) Use L'Hôpital's rule to evaluate the limit.

Note: you may need to apply L'Hôpital's rule multiple times in order to convert the limit into a form that is easy to evaluate. You must check *each time* that it is one of the two allowable indeterminate forms.

1. $\lim_{x\to 2} \frac{(3x+2)^{1/3}-2}{x-2}$
2. $\lim_{x\to\infty} \frac{4x^3-2x^2+6}{\pi x^3+4}$
3. $\lim_{x\to\pi/2} \frac{2\tan x}{\sec^2 x}$
4. $\lim_{x\to 0} \frac{\tan 4x}{\tan 7x}$
5. $\lim_{x\to 0} \frac{1-\cos 3x}{x^2}$
6. $\lim_{x\to 1} \frac{x^n-1}{x-1}$ (n is a positive integer)
7. Evaluate the following limit:
$$\lim_{x\to 1^-}(1-x)\tan\left(\frac{\pi x}{2}\right).$$
Hint: rewrite the *product* as a *quotient* and apply L'Hôpital's rule. Make sure you verify that you have one of the two acceptable indeterminate forms before applying the rule.
8. Evaluate:
$$\lim_{x\to\infty}(x-\sqrt{x^2+1}).$$
Hint: write as $\frac{x-\sqrt{x^2+1}}{1}$ and rationalize the numerator.
9. Evaluate:
$$\lim_{x\to 0^+}\left(\cot x-\frac{1}{x}\right).$$
Hint: write $\cot x$ as $\cos x/\sin x$, then find the common denominator with $1/x$ and write as a single fraction.

10. Let $f(x) = x^{2x}$. We will evaluate $\lim_{x\to 0^+} f(x)$.

   (a) Let $g(x) = \ln f(x)$.

   (b) Rewrite $g(x)$ as a quotient and use L'Hôpital's rule to evaluate $\lim_{x\to 0^+} g(x)$.

   (c) Since e^x is a continuous function,

   $$\lim_{x\to 0^+} e^{g(x)} = e^{\lim_{x\to 0^+} g(x)}.$$

   Use this fact to evaluate $\lim_{x\to 0^+} f(x)$.

11. We say that $g(x)$ *grows faster than* $f(x)$ if

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0.$$

Use L'Hôpital's rule to show that 1.000001^x grows faster than x^{10000}. Why do you think people use the term "exponential" to refer to quantities that grow very rapidly?