Increasing & decreasing functions; first derivative test for local extrema

TA: Sam Cole

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1. Suppose f is defined on $[a, b]$ and c is somewhere in the interval $[a, b]$ (so $a < c < b$). Suppose f is *increasing* on the interval $[a, c]$ and *decreasing* on $[c, b]$.
   (a) Sketch the graph of such a function on the interval $[a, b]$.
   (b) What can you say about the point $x = c$?

2. The following is a precise definition of local minimum:

   Definition 1. *$f(x)$ has a* local minimum *at $x = c$ if there is an interval $[a, b]$ containing c such that $f(c) \leq f(x)$ for all x in $[a, b]$.*

   According to the above definition, does a constant function (i.e., a horizontal line) have any local minima? If so, where?

3. Sketch the graph of a single function $f(x)$ that satisfies all of the following:
   - $f(x)$ is continuous on $(-\infty, \infty)$.
   - $f'(x) < 0$ on $(-\infty, 2)$.
   - $f'(x) > 0$ on $(2, 5)$.
   - $f'(x) < 0$ on $(5, \infty)$.

For problems 4-10, do the following:

(a) Find the critical points of f.

(b) Determine the intervals on which f is increasing/decreasing (be mindful of the domain of f).

(c) Determine whether each critical point is a local minimum, local maximum, or neither.

4. $f(x) = x^2 + 3$
5. $f(x) = x\sqrt{9 - x^2}$
6. $f(x) = 2x^3 + 3x^2 - 12x + 1$
7. $f(x) = \frac{x^2}{x^2 - 1}$
8. $f(x) = \arctan x - x^3$
9. $f(x) = xe^{-x}$
10. $f(x) = x^2 - 2\ln x$