

Derivatives of logs and inverse trig functions

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1. What is the derivative of $f(x) = \log_b(x)$? Hint: use the change of base formula to convert to \ln .

Differentiate the following functions by using the following steps:

- (a) Let $g(x) = \ln(f(x))$.
- (b) Use the chain rule to find a generic formula for $g'(x)$ in terms of x , $f(x)$, and $f'(x)$.
- (c) Now find $g(x)$ for this specific problem. Use rules for logarithms to simplify as much as possible.
- (d) Find $g'(x)$ using your answer to part (c).
- (e) You now have two formulas for $g'(x)$ (from parts (b) and (d)). Set them equal and solve for $f'(x)$!

2. $f(x) = \frac{(x+1)^3(x-4)^5}{(5x+3)^2}$

3. $f(x) = x^{\ln x}$

4. $f(x) = \left(1 + \frac{1}{x}\right)^{2x}$

5. Let $f(x) = \cos x$. We will derive the formula for the derivative of $f^{-1}(x) = \arccos x$.

- (a) What is $f'(x)$?
- (b) Recall that $(f^{-1})'(x) = 1/f'(f^{-1}(x))$ (see 10/7 handout on implicit differentiation). Plug in $\cos x$ and its derivative.
- (c) Draw a right triangle in which the hypotenuse has length 1 and one leg has length x .
- (d) What is the length of the other leg?
- (e) In your drawing, indicate which angle represents $\arccos x$.
- (f) From your drawing, what is the sine of this angle?
- (g) Plug your answer for (f) into your answer for (b).

So now in case you forget your inverse trig derivatives during midterm 2 you can just figure them out yourself!

Differentiate the following

6. $f(x) = \arccos(1/x)$

7. $f(x) = (\arccos x)^2$

8. $f(x) = \sin(\arccos(2x))$

9. Apply the technique from problem 5 to find the derivatives of $\arcsin x$ and $\arctan x$.

10. If you're feeling adventurous, try $\sec^{-1} x$, $\csc^{-1} x$, and $\cot^{-1} x$.