Mean value theorem

TA: Sam Cole

11/6/14

1. State the mean value theorem (try not to look it up online).
2. Recall that the slope of the secant line of a function f on the interval $[a, b]$ is given by

$$m_{\text{sec}} = \frac{f(b) - f(a)}{b - a}.$$

   (a) Rephrase the mean value theorem in terms of slopes of tangent and secant lines.

   (b) Sketch a function f satisfying the conditions of the mean value theorem on the interval $[a, b]$. You don't need to define $f(x)$—just draw a picture.

   (c) Draw the secant line on $[a, b]$.

   (d) Try to find the point c that is guaranteed to exist by the MVT in your graph (just eyeball it). Draw the tangent line at that point.

3. State Rolle's theorem. Explain why it's just a special case of the MVT. Draw a picture of what's happening, as in problem 2.

Problems 4-8:

(a) Are the conditions of Rolle's theorem satisfied?

(b) If so, find the point that is guaranteed to exist by Rolle's theorem.

4. $f(x) = x(x-1)^2; [0, 1]$
5. $f(x) = \sin(2x); [0, \pi/2]$
6. $f(x) = \cos(4x); [\pi/8, 3\pi/8]$
7. $f(x) = 1 - |x|; [-1, 1]$
8. $f(x) = x^3 - 2x^2 - 8x; [-4, 2]$

Problems 8-13: same as 4-8, but with the MVT. Warning: your answers may not be nice!

8. $f(x) = 7 - x^2; [-1, 2]$
9. $f(x) = e^x; [0, \ln 4]$
10. $f(x) = x + 1/x; [1, 3]$
11. $f(x) = 2x^{1/3}; [-8, 8]$
12. $f(x) = x/(x+2); [-1, 2]$
13. $f(x) = \ln(2x); [e, e^2]$