Minima and maxima on a closed interval

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1. Sketch a function defined on $[0, 1]$ which has an absolute maximum at $x = \frac{1}{2}$.
2. Sketch a function defined on $[0, 1]$ which has an absolute maximum at $x = \frac{1}{2}$ and $f'(\frac{1}{2})$ is undefined.
3. Sketch a function defined on $[0, 1]$ which has an absolute maximum at $x = 0$, an absolute minimum at $x = \frac{1}{2}$, and a local maximum at $x = \frac{3}{4}$.
4. Sketch a function f which has *no* maximum (local or absolute) on $[0, 1]$. Hint: f need not be continuous.
5. True or false: if f has a *local* minimum at $x = c$, then it has an *absolute* minimum at $x = c$. If false, sketch a counterexample.
6. True or false: if f has an *absolute* minimum at $x = c$, then it has a *local* minimum at $x = c$. If false, sketch a counterexample.
7. True or false: if f has a local minimum/maximum at $x = c$, then $f'(c) = 0$ or $f'(c)$ does not exist. If false, sketch a counterexample.
8. True or false: if $f'(c) = 0$, then f has a local minimum/maximum at $x = c$. If false, sketch a counterexample.
9. The following statement is *false*: if f is defined on $[a, b]$, then it has a global minimum *and* maximum somewhere on $[a, b]$. What condition on f would make the statement true?
10. Find the absolute minimum and maximum of $f(x) = x^2 - 10$ on the interval $[a, b] = [-2, 3]$ by doing the following:
    (a) Find $f'(x)$.
    (b) Find all *critical points* of f on $[a, b]$. A critical point is a point where $f'(x) = 0$ or $f'(x)$ does not exist.
    (c) Evaluate $f(x)$ at each critical point *and* the endpoints of the interval. The largest value you get is the absolute maximum, and the smallest value you get is the absolute minimum.

    Note that the above steps work for any f that is continuous on $[a, b]$.