Related rates; optimization

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1 Related rates

1. The edges of a cube are increasing at a rate of 2 cm/s. How fast is the volume increasing when the length of each edge is 50 cm?

   Suggested steps:

   (a) Let V be the volume of the cube and s the length of a side. Your goal is to find $\frac{dV}{dt}$

   (b) Write an equation relating V and s.

   (c) Differentiate both sides *with respect to* t to get a formula for $\frac{dV}{dt}$ in terms of s and $\frac{ds}{dt}$. Don't forget to use the chain rule! This step *relates* the rates of change of volume and side length, hence the term *related rates*.

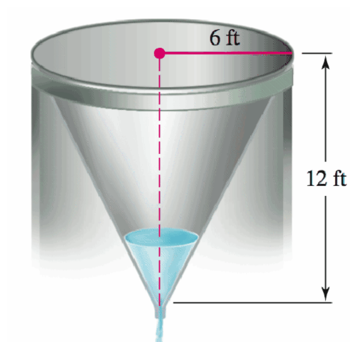
   (d) What is $\frac{ds}{dt}$? Hint: it's given to you.

   (e) Plug in $s = 50$ and your answer to (d) to your formula for $\frac{dV}{dt}$ to get the answer.

2. A spherical balloon is increasing in volume at a rate of 15 in^3/min. What is the rate of change of the radius of the balloon when the radius is 10 in?

3. A 5-ft-tall woman walks 8 ft/s toward a street light that is 20 ft tall. What is the rate of change of the length of her shadow when she is 15 ft from the street light?

4. A conical water tank with height 12 ft and radius 6 ft (see picture) is draining at a rate of 2 ft^3/s. What is the rate of change of the depth of the water when the depth is 3 ft? (Hint: use similar triangles.)

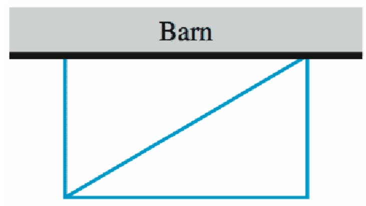


2 Optimization

1. What is the maximum area of a rectangle with perimeter 10 m? What are the dimensions of such a rectangle?

2. What is the smallest possible sum of two real numbers whose product is 50? What are the two numbers?

3. A farmer has 200 m of fencing available and wants to build 2 triangular pens against a barn, as shown:



   (a) Let l and w be the dimensions of the rectangle. What is the total length of the fence in terms of l and w?

   (b) What dimensions of the rectangle give the maximum area of the pens?

4. An *open* cylindrical can (no top) has volume

   (a) Let l, w, and h be the dimensions of the box. What is the surface area of the outside of the box in terms of l, w, and h?

   (b) What is the minimum possible surface area of the box?

5. A rectangle is constructed with its base on the x-axis and two of its vertices on the parabola $y = 16 - x^2$. What is the maximum possible area of such a rectangle? What dimensions give this area?

   Suggested steps:

   (a) Draw a picture!

   (b) Let (x, y) be one of the vertices on the parabola. What are the dimensions of the rectangle in terms of x and y? Are there any constraints on x and y?

   (c) What is the area of the rectangle in terms of x and y?

   (d) Since (x, y) is a point on the parabola, you can substitute $y = 16 - x^2$. Now maximize your area function from part (c).