PBR vs. Keystone Light
An application of differentials

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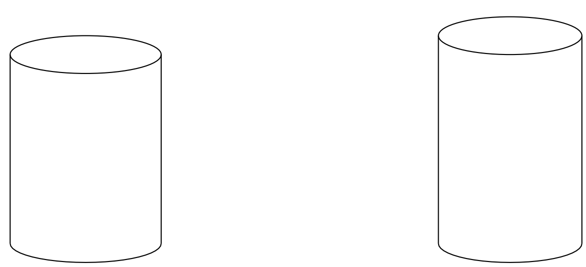
If you're not familiar with cheap beer, most canned beers, including Pabst Blue Ribbon (PBR), are sold in 12-oz cylindrical cans with radius ≈ 1 in and height ≈ 5 in. Some, however, such as Keystone Light, come in slightly taller, thinner cans (but still have a volume of 12 oz). This is a marketing ploy used by the company that produces Keystone: a can of Keystone "looks bigger" than a can of PBR because it's somewhat taller but only slightly thinner. So what's the deal?



(a) A can of PBR (b) A can of Keystone Light

Figure 1: Which can appears to be bigger?

Recall that the volume of a cylinder is given by

$$V(r,h) = \pi r^2 h.$$

Then the partials of $V(r,h)$ are

$$V_r(r,h) = 2\pi r h, V_h(r,h) = \pi r^2.$$

Now imagine that you have a standard size beer can ($r = 1, h = 5$) and you want to vary the dimensions slightly while keeping the volume the same. The equation relating the differentials dV, dr, dh is

$$\begin{aligned} dV &= V_r(1,5)dr + V_h(1,5)dh \\ &= 10\pi dr + \pi dh. \end{aligned}$$

So we see that volume is 10 times more sensitive to changes in radius than changes in height! Therefore, if we want to keep volume the same, we have

$$0 = \Delta V \approx 10\pi\Delta r + \pi\Delta h,$$

hence

$$\Delta h \approx -10\Delta r.$$

Thus, if we *decrease* the radius by ϵ (i.e., $\Delta r = -\epsilon$) it allows us to *increase* the height by about 10ϵ without changing the volume. For example, if we decrease the radius by just 1/20 in, which is barely noticeable, we can increase the height by about 1/2 in, which is very noticeable! The result is a can with radius .95 in and height 5.5 in, which will appear to be about the same radius as a PBR can but 1/2 in taller!

This example was adapted from a very nice book by Susan Colley [1], who was my calc. 3 professor when I was an undergrad.

References

[1] Susan Jane Colley. *Vector Calculus.* Pearson, 4th edition, 2011.