Properties of vectors cheat sheet

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Definition 1 (algebraic) *A* vector *in* $\mathbb{R}^2$ *is an ordered pair* $\mathbf{u} = \langle x, y \rangle$, *where* x *and* y *are real numbers.*

Definition 2 (geometric) *A* vector *in* $\mathbb{R}^2$ *is an object in the plane with* magnitude *(length) and* direction.

In the following table, let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors, let c be a scalar (i.e., a real number), and let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ be points.

	Algebraic interpretation	**Geometric interpretation**
$\mathbf{u} = \mathbf{v}$	Corresponding components are equal, i.e. $u_1 = v_1$ and $u_2 = v_2$.	$\mathbf{u}$ and $\mathbf{v}$ have same magnitude and direction.
$\mathbf{0}$	$\langle 0, 0 \rangle$	Vector with magnitude 0 and no direction
$\lvert\mathbf{u}\rvert$	$\sqrt{u_1^2 + u_2^2}$	Magnitude of $\mathbf{u}$ (length of arrow); distance from (u_1, u_2) to origin
$c\mathbf{u}$	$\langle cu_1, cu_2 \rangle$	Scale $\mathbf{u}$ by a factor of $\lvert c\rvert$; reverse direction if $c < 0$
$-\mathbf{u}$	$-1\mathbf{u} = \langle -u_1, -u_2 \rangle$	Reverse direction of $\mathbf{u}$, i.e. rotate it $180°$ about its tail
Parallel vectors	$\mathbf{u} = c\mathbf{v}$ for some c.	$\mathbf{u}$ and $\mathbf{v}$ are parallel; $\mathbf{u}$ and $\mathbf{v}$ have equal or opposite directions; $\mathbf{u}$ and $\mathbf{v}$ lie on the same line if they are both drawn in standard position.
$\mathbf{u} + \mathbf{v}$	$\langle u_1 + v_1, u_2 + v_2 \rangle$	Triangle rule; parallelogram rule
$\mathbf{u} - \mathbf{v}$	$\mathbf{u} + (-\mathbf{v}) = \langle u_1 - v_1, u_2 - v_2 \rangle$	Apply triangle rule or parallelogram rule to $\mathbf{u}$ and $-\mathbf{v}$.
$\overrightarrow{PQ}$	$\langle x_2 - x_1, y_2 - y_1 \rangle$	Vector from P to Q

See section 11.1 of Briggs & Cochran for details.