110 Facts About Binary Numbers

1. If the last digit of a binary number is 1, the number is odd; if it’s 0, the number is even.
   Ex: 1101 represents an odd number (13); 10010 represents an even number (18).

2. To convert a binary number to base $2^k$, split it into groups of $k$ digits (adding leading 0s if necessary), then convert each group to base $2^k$.
   Ex: Convert the number 1001011111 to base 8.
   First, note that $8 = 2^3$, so we should split the number into groups of 3 digits:
   
   $001 \mid 001 \mid 011 \mid 111$

   Note that we added two leading 0s to make the number of digits a multiple of 3. Next, we convert each group of 3 digits to base 8:

   $001 \mid 001 \mid 011 \mid 111$

   $1 \mid 1 \mid 3 \mid 7$

   Thus, the number in base 8 is 1137.

3. In a base-$n$ representation of a number, no digit exceeds $n - 1$.
   Ex: Every digit of a base 3 number must be 0, 1, or 2.

4. In an $n$-bit, unsigned binary system, the largest number that can be represented is all 1s and the smallest number is all 0s. These numbers represent $2^n - 1$ and 0, respectively.
   Ex: In an 8-bit, unsigned binary system, the largest number that can be represented is 11111111 = $2^8 - 1 = 255$, and the smallest is 00000000 = 0.

5. In an $n$-bit, signed, two’s complement binary system, the largest number that can be represented is a 0 followed by all 1s, and the smallest is a 1 followed by all 0s. These numbers represent $2^{n-1} - 1$ and $-2^{n-1}$, respectively.
Ex: In an 8-bit, signed, two’s complement binary system, the largest number that can be represented is \(01111111 = 2^7 - 1 = 127\), and the smallest is \(10000000 = -2^7 = -128\).

6. In an \(n\)-bit, signed, two’s complement binary system, a negative number \(x\) is the same as the positive number \(2^{n-1} + x\), except the leading (leftmost) bit is 1 instead of 0. Therefore, you can find the two’s complement representation of \(x\) by adding \(2^{n-1}\), finding the \(n\)-bit unsigned representation, and changing the first bit to a 1.

**Ex:** In an 8-bit, signed binary system, find the representation of \(-54\).
First, we find the representation of \(2^7 + (-54) = 128 - 54 = 124\); it is 01111100. Thus, \(-54\) is 11111100.

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\begin{align*}
128 & : 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \\
-54 & : 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0
\end{align*}
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(Image taken from XKCD)