110 Facts About Binary Numbers

1. If the last digit of a binary number is 1, the number is odd; if it's 0, the number is even.

   Ex: 1101 represents an odd number (13); 10010 represents an even number (18).

2. To convert a binary number to base 2^k, split it into groups of k digits (adding leading 0s if necessary), then convert each group to base 2^k.

   Ex: Convert the number 1001011111 to base 8.

   First, note that $8 = 2^3$, so we should split the number into groups of 3 digits:

   $$001 \mid 001 \mid 011 \mid 111$$

   Note that we added two leading 0s to make the number of digits a multiple of 3. Next, we convert each group of 3 digits to base 8:

   $$\begin{array}{c|c|c|c} 001 & 001 & 011 & 111 \\ 1 & 1 & 3 & 7 \end{array}$$

   Thus, the number in base 8 is 1137.

3. In a base-n representation of a number, no digit exceeds $n - 1$.

   Ex: Every digit of a base 3 number must be 0, 1, or 2.

4. In an n-bit, unsigned binary system, the largest number that can be represented is all 1s and the smallest number is all 0s. These numbers represent $2^n - 1$ and 0, respectively.

   Ex: In an 8-bit, unsigned binary system, the largest number that can be represented is $11111111 = 2^8 - 1 = 255$, and the smallest is $00000000 = 0$.

5. In an n-bit, *signed*, two's complement binary system, the largest number that can be represented is a 0 followed by all 1s, and the smallest is a 1 followed by all 0s. These numbers represent $2^{n-1} - 1$ and -2^{n-1}, respectively.

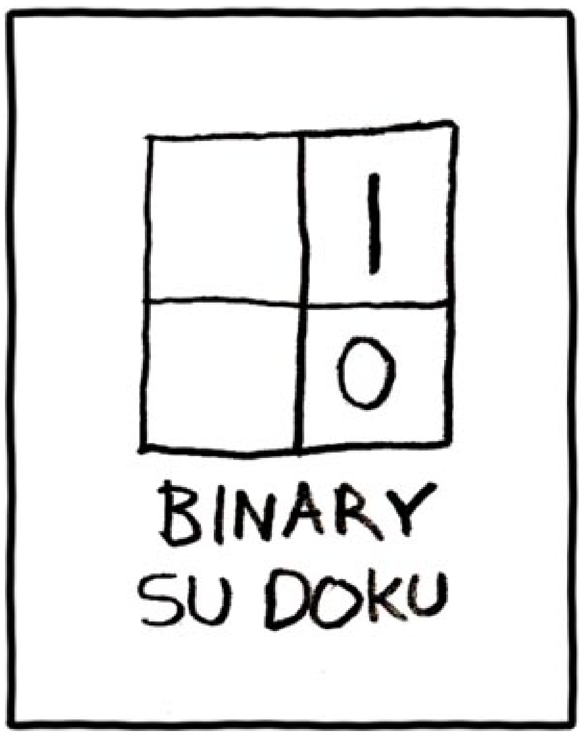
Ex: In an 8-bit, signed, two's complement binary system, the largest number that can be represented is $01111111 = 2^7 - 1 = 127$, and the smallest is $10000000 = -2^7 = -128$.

6. In an n-bit, signed, two's complement binary system, a negative number x is the same as the positive number $2^{n-1} + x$, except the leading (leftmost) bit is 1 instead of 0. Therefore, you can find the two's complement representation of x by adding 2^{n-1}, finding the n-bit unsigned representation, and changing the first bit to a 1.

   Ex: In an 8-bit, signed binary system, find the representation of -54.

   First, we find the representation of $2^7 + -54 = 128 - 54 = 124$; it is 01111100. Thus, -54 is 11111100.

$$\begin{array}{r@{\;}cccccccc} 128: & \mathbf{0} & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ -54: & \mathbf{1} & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{array}$$



(Image taken from XKCD)