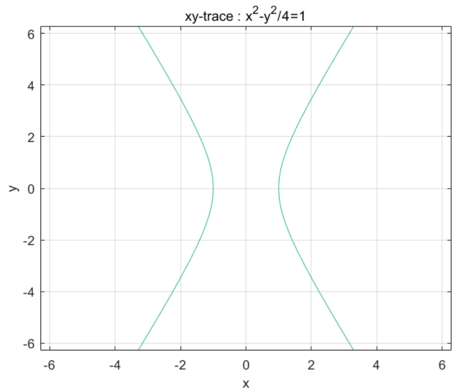
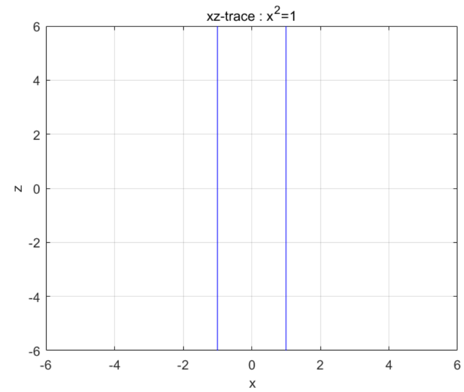
Worksheet Solution - Week 5

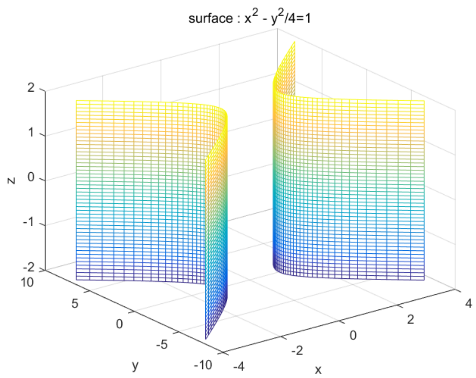
1. For the following surfaces, find $xy-, yz-$, and $xz-$traces, and sketch a surface.

   (a) $x^2 - \frac{y^2}{4} = 1$

   Proof. yz-trace does not exist for this problem.



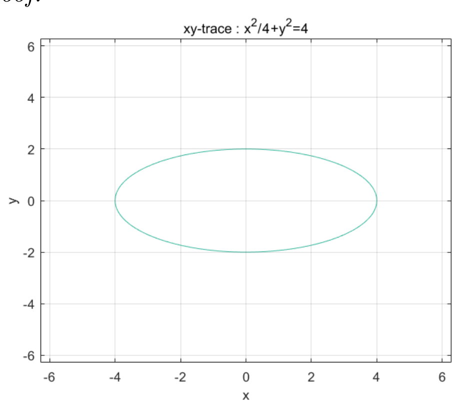


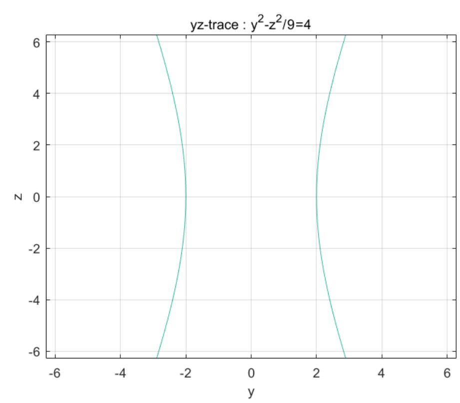


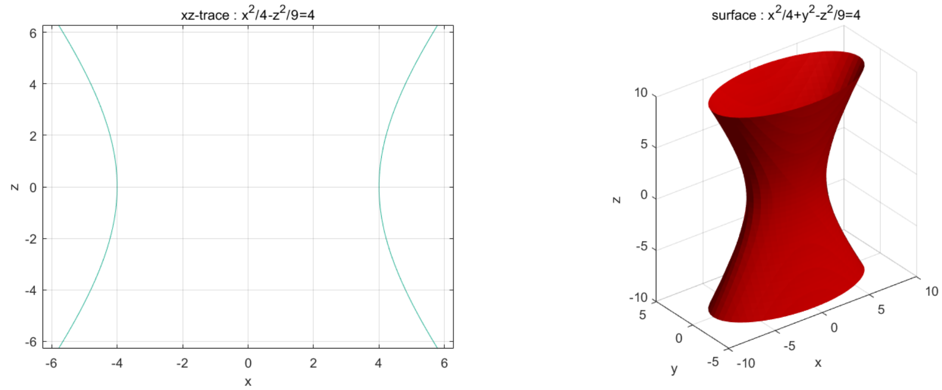
□

   (b) $\frac{x^2}{4} + y^2 - \frac{z^2}{9} = 4$

   Proof.







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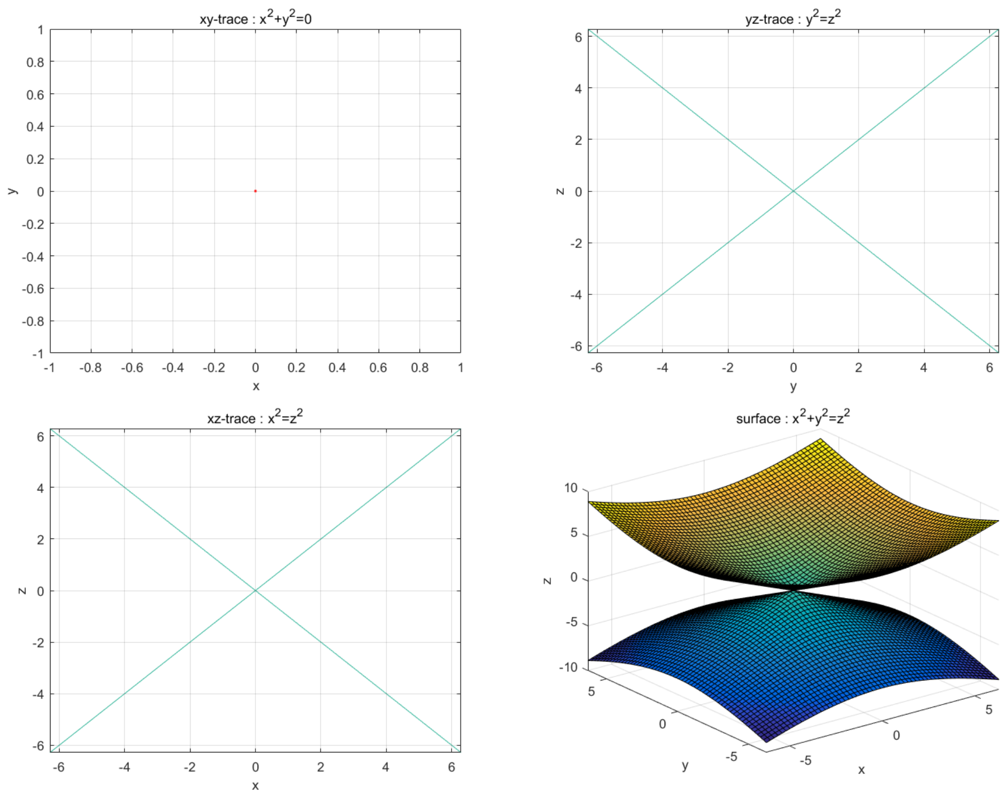
(c) $x^2 + y^2 = z^2$

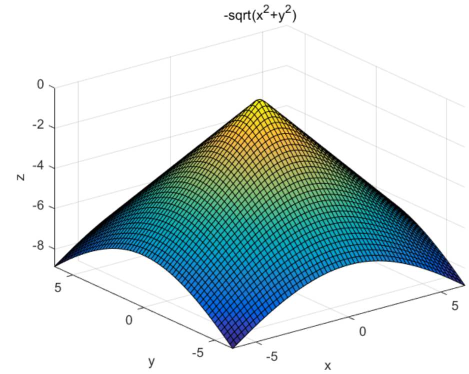
Proof.



$\square$

2. Sketch the surface defined by $z = -\sqrt{x^2 + y^2}$. What is different from the surface defined by $z^2 = x^2 + y^2$?

Proof.
For the case of $z^2 = x^2 + y^2$, this graph looks like a hourglass. However, for the case of $z = -\sqrt{x^2 + y^2}$ this graph shows only down part of this hourglass. (sketch is in the next page)

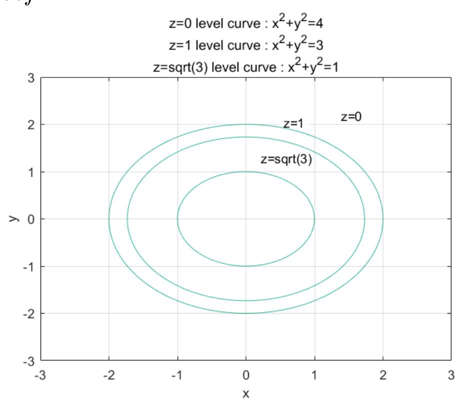


□

3. Graph several level curves of the following functions using the given window. Label at least two level curves with their z-values.

   (a) $z = \sqrt{4 - x^2 - y^2}$; $[-3, 3] \times [-3, 3]$

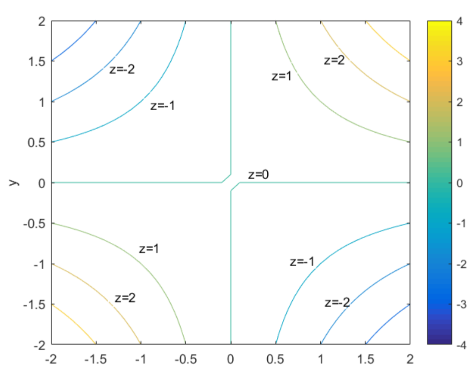
   Proof.



□

   (b) $z = xy$; $[-2, 2] \times [-2, 2]$

   Proof.



□

4. Calculate the limit of multivariable functions or show that the limit does not exist.

   (a) $\lim_{(x,y)\to(1,2)} \frac{4x^2-4xy+y^2}{2x-y}$

   Proof.
   $$\lim_{(x,y)\to(1,2)} \frac{4x^2-4xy+y^2}{2x-y} = \lim_{(x,y)\to(1,2)} \frac{(2x-y)^2}{2x-y} = \lim_{(x,y)\to(1,2)} (2x-y) = 0$$ □

(b) $\lim_{(x,y)\to(0,0)} \frac{x^2+3y^4}{x-y^4}$

Proof. Use the Two path test.
(i) Take the path along x-axis.
In this case, we could take $(x, y) = (t, 0)$, and we can change the $\lim_{(x,y)\to(0,0)}$ into $\lim_{t\to0,(x,y)=(t,0)}$. With these tools, we can compute the limit of the function along x-axis as following :

$$\lim_{t\to0,(x,y)=(t,0)} \frac{x^2+3y^4}{x-y^4} = \lim_{t\to0} \frac{t^2+3\cdot0^4}{t-0^4} = \lim_{t\to0} t = 0$$

(ii) Take the path along y-axis.
In this case, we could take $(x, y) = (0, t)$, and we can change the $\lim_{(x,y)\to(0,0)}$ into $\lim_{t\to0,(x,y)=(0,t)}$. With these tools, we can compute the limit of the function along x-axis as following :

$$\lim_{t\to0,(x,y)=(0,t)} \frac{x^2+3y^4}{x-y^4} = \lim_{t\to0} \frac{0^2+3t^4}{0-t^4} = \lim_{t\to0} \frac{3t^4}{-t^4} = -3$$

Since two values 0 and -3 are not the same value, by using two path test, we can conclude that this limit does not exist. □

(c) $\lim_{(x,y,z)\to(-1,1,1)} \frac{xz+2x+y^2z+2y^2}{x+y^2}$

Proof. $\lim_{(x,y,z)\to(-1,1,1)} \frac{xz+2x+y^2z+2y^2}{x+y^2} = \lim_{(x,y,z)\to(-1,1,1)} \frac{(z+2)(x+y^2)}{x+y^2} = \lim_{(x,y,z)\to(-1,1,1)} (z+2) = 3$ □