

# Math 215 - Homework 1

Due Friday, September 7

1. Using a truth table, prove that **De Morgan's Laws** are tautologies:

(a)  $\neg(P \wedge Q) \leftrightarrow \neg P \vee \neg Q$

(b)  $\neg(P \vee Q) \leftrightarrow \neg P \wedge \neg Q$

2. Decide whether each of the following is a tautology, contradiction, or neither.

(a)  $P \rightarrow \neg P$

(b)  $(P \wedge Q) \vee (\neg P \wedge \neg Q)$

(c)  $P \rightarrow (P \rightarrow (P \rightarrow Q))$

(d)  $P \leftrightarrow [P \wedge (P \vee Q)]$

(e)  $(P \wedge \neg Q) \wedge (P \rightarrow Q)$

3. Give a useful denial of each of the following assertions:

(a) I like dessert but can't have ice cream.

(b)  $x < y$  or  $m^2 < 1$ .

(c) We have to cancel the trip if the weather hasn't improved.

(d)  $n$  is an odd multiple of 5.

4. Show that if  $a, b, c$  are positive integers,  $a$  divides  $b$ , and  $b$  divides  $c$ , then  $a$  divides  $c$ .

5. Prove that if  $a, b$  are positive integers,  $a$  divides  $b$ , and  $b$  divides  $a$ , then  $a = b$ .