Math 215 - Homework 1 Grading Guidelines

Due Friday, September 7

1. Using a truth table, prove that **De Morgan's Laws** are tautologies:

(a) $\neg(P \wedge Q) \leftrightarrow \neg P \vee \neg Q$

(b) $\neg(P \vee Q) \leftrightarrow \neg P \wedge \neg Q$

1 point. Don't need to check in detail, but no credit if any "F" appears in the column of De Morgan's Law.

2. Decide whether each of the following is a tautology, contradiction, or neither.

(a) $P \to \neg P$

(b) $(P \wedge Q) \vee (\neg P \wedge \neg Q)$

(c) $P \to (P \to (P \to Q)))$

(d) $P \leftrightarrow [P \wedge (P \vee Q)]$

(e) $(P \wedge \neg Q) \wedge (P \to Q)$

5 points, 1 per part. Full or no credit depending on whether the answer is correct.

3. Give a useful denial of each of the following assertions:

(a) I like dessert but can't have ice cream.
I don't like dessert or can have ice cream.

(b) $x < y$ or $m^2 < 1$.
$x \geq y$ and $m^2 \geq 1$.

(c) We have to cancel the trip if the weather hasn't improved.
The weather hasn't improved, and we don't have to cancel the trip.

(d) n is an odd multiple of 5.
n is even or not a multiple of 5. (Accept "n is not an odd multiple of 5.")

4 points (1 per part). All or no credit ("and", "or", "not" placements are important here)

4. Show that if a, b, c are positive integers, a divides b, and b divides c, then a divides c.

5 points.

Notes: It is important that k and l are some *integers* whose existence follows from definition of "divides", and that k and l are *not* assumed to be equal. Students should be discouraged from using the standard fraction notation here ("$\frac{a}{b}$") as this is very different from the notion "a divides b" (written $a|b$)

Suppose a divides b and b divides c, a, b, c positive integers. Then there are integers k and l so that $a \cdot k = b$ and $b \cdot l = c$. Substituting,

$$c = b \cdot l = (a \cdot k) \cdot l = a \cdot (k \cdot l).$$

Since $c = a \cdot (k \cdot l)$ and $k \cdot l$ is an integer, we have a divides c.

5. Prove that if a, b are positive integers, a divides b, and b divides a, then $a = b$.

5 points.

Assuming a and b divide one another, we have k and l such that $a \cdot k = b$ and $b \cdot l = a$. Substituting, $b = a \cdot k = (b \cdot l) \cdot k = b \cdot (k \cdot l)$.

Now, since b is positive we have $b \neq 0$ and can multiply both sides by b^{-1} and obtain $k \cdot l = 1$. By uniqueness of additive inverse, $k = l^{-1}$. Now k is a positive integer, thus $k \geq 1$. But then $k = 1$; since $k > 1$ would imply $l < 1$ (Worksheet 1), and l is also a positive integer. So we must have $k = 1$. This shows $b = a \cdot k = a \cdot 1 = a$, as claimed.

Notes: Same remarks as above apply.

It is important that they invoke positivity of b if they multiply through by b^{-1} as was done here.

I would like some justification for why $l < 1$ such as citing the worksheet or the book.

I told students they would not be penalized for failing to write in complete sentences, but feel free to deduct points if their inferences are at any point remotely unclear.