Math 215 - Homework 2

Due Friday, September 21

1. For each of the following, give the statement's contrapositive and its converse. For each, either state that it is true or give a counterexample to show it is not.

(a) If n is an even integer, then n is not prime.

(b) If $f(x)$ is differentiable at $x = 7$, then $f(x)$ is continuous at $x = 7$.

(c) If n is odd, then either $n + 1$ or $n - 1$ is divisible by 6.

2. Prove by contradiction that if a, b are real numbers with $a \cdot b = 0$, then $a = 0$ or $b = 0$.

3. Show for all positive integers n that 4 divides $5^n + 7$.

4. Prove that for all positive integers n,

$$\sum_{i=1}^{n} i^2 = \frac{2n^3 + 3n^2 + n}{6}.$$

5. Let k, n be positive integers. Prove that if k is the least integer divisor of n such that $k > 1$, then k is prime.

6. Show that an integer $n > 1$ is prime if and only if n has no prime divisors p with $p^2 \leq n$.