Math 215 - Homework 4

Due Friday, October 19

1. For each of the following sets A_i, give three examples of elements A_i, and three examples of elements of A_i^c.

(a) $A_1 = \{a \in \mathbb{Z} \mid \sqrt{a} \in \mathbb{Z}\}$

(b) $A_2 = \{p^q \mid p, q \in \mathbb{N} \text{ and } p, q \text{ are primes}\}$

(c) $A_3 = \{x \in \mathbb{R} \mid \text{for some } n \in \mathbb{Z},\ 3n \leq x < 3n + 2\}$

(d) $A_4 = \{\langle x, y\rangle \in \mathbb{N} \times \mathbb{R} \mid e^y = 2x\}$

2. Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{Z}$ be defined by $f(x, y) = x - y$.

(a) Give a table of values for $f|\{0, 1, 2, 3\}^2$.

(b) Is f injective? Surjective? Prove your answers.

3. Let A, B be sets such that $|A| = 7$, $|B| = 29$, and $|A \cup B| = 32$. Determine the sizes of each of the following sets:

(a) $A \cap B$

(b) $B - A$

(c) $A \times B$

(d) $(A \times A) - \{\langle a, b\rangle \in A^2 \mid a = b\}$

4. Let $f : A \to B$ be a function. Show that f is injective if and only if for all sets $X, Y \subseteq A$,

$$\mathrm{Im}(f|(X \cap Y)) = \mathrm{Im}(f|X) \cap \mathrm{Im}(f|Y).$$

5. Recall $\mathbb{R}^{\geq 0} = \{x \in \mathbb{R} \mid x \geq 0\} = [0, \infty)$. For each of the following, give an example of a function $f_i : \mathbb{R}^{\geq 0} \to \mathbb{R}$ with the listed properties.

(a) f_1 is injective, but not surjective.

(b) f_2 is surjective, but not injective.

(c) f_3 is bijective.

6. Let $g : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be defined by

$$g(x, y) = (x + y + 1)(x + y) + 2y.$$

(a) Give a table of values for $g|\{0, 1, 2, 3\}^2$.

(b) Is g injective? Surjective? Prove your answers.