Math 215 - Homework 7

Due Wednesday, December 5

1. Suppose $X \subseteq Y$ and X is uncountable. Show Y is uncountable.

2. Suppose there is an injection $f : \mathbb{R} \to X$. Show X is uncountable.

3. Suppose there is a surjection $f : \mathbb{N} \to X$. Show X is countable.

Let A be a set. For the next problems, we let $A^{\mathbb{N}}$ denote the set of infinite sequences in A,

$$A^{\mathbb{N}} := \mathrm{Fun}(\mathbb{N}, A),$$

and $A^{<\mathbb{N}}$ denotes the set of finite sequences in A,

$$A^{<\mathbb{N}} = \bigcup_{k \in \mathbb{N}} \mathrm{Fun}(\mathbb{N}_k, A).$$

4. Show $\mathbb{N}^{<\mathbb{N}}$ is countable.

5. Use the Cantor-Shröder-Bernstein Theorem to show $|\mathbb{R}| = |\{0, 1\}^{\mathbb{N}}|$.