Math 215 - Review for Midterm 1

September 27, 2018

1. Give a truth table for each of the following propositional forms:

(a) $P \to (P \vee Q)$

(b) $P \wedge (P \leftrightarrow \neg Q)$

(c) $(P \vee Q) \to \neg R$

Are any of these tautologies? Contradictions?

2. Prove by contradiction that if xy is even, then x is even or y is even.

3. Show that if $x \in \mathbb{R}$ with $x \geq -1$, then for all $n \in \mathbb{N}$,

$$1 + nx \leq (1+x)^n.$$

Where did you use the assumption that $x \geq -1$?

4. Show that for all integers $n \geq 1$:

$$\sum_{i=1}^{n} 2i - 1 = n^2.$$

5. Show that for all sets A, B, if $A \subseteq B$ then $B^c \subseteq A^c$.

6. Let A_1, A_2, A_3 be sets. Show there are sets B_1, B_2, B_3 such that for i with $1 \leq i \leq 3$, we have $B_i \subseteq A_i$, $A_1 \cup A_2 \cup A_3 = B_1 \cup B_2 \cup B_3$ and the B_i are *disjoint*: that is, $B_i \cap B_j = \varnothing$ whenever $i \neq j$.

7. Give a translation of each of the following in plain English; determine whether each is true or false.

(a) $\forall x \in \mathbb{R}\ \exists y \in \mathbb{N}\ x < y$

(b) $\exists y \in \mathbb{Z}\ \forall z \in \mathbb{R}\ y < z \cdot z$

(c) $\exists x \in \mathbb{R}\ \exists y \in \mathbb{R}\ y \neq 0 \wedge x + y = x$

(d) $\forall x \in \mathbb{N}\ \forall y \in \mathbb{N}\ \exists z \in \mathbb{N}\ x < y \to (x < z \wedge z < y)$

8. Using mathematical symbols ($+$, $\cdot$, quantifiers, etc.) only, give a definition of the predicate $P(n)$: "n is prime."

9. The *twin prime conjecture* states: "there are infinitely many primes p such that $p + 2$ is also prime."
Using symbols, give a statement of the twin prime conjecture. (You may use $P(n)$ as an abbreviation for the predicate you gave in the previous problem.)